4. Emergence of Computational Fluid Dynamics

The 4th of 12 lectures by Prof. Raj to share his perspective on effective application of computational aerodynamics to aircraft design.

Each lecture contains excerpts from the presentation shown below describing his exciting journey on a long and winding road for more than five decades!

Reflections on the Effectiveness of Applied Computational Aerodynamics for Aircraft Design

https://www.aoe.vt.edu/people/emeritus/raj/personal-page/reflections-on-ACA-effectiveness.html

Pradeep Raj, Ph.D.
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Advanced Development Programs

LOCKHEED MARTIN

Lecture 3: Key Takeaways

- **1755-57: The Euler Equations for inviscid, compressible flows**
  - Euler derived three *equations of motion* from the *first axioms of mechanics* which, combined with continuity equation and equation of state, gave *"…five equations encompassing the entire theory of the motion of fluids."*
  - Solving the equations was hampered by *"…the Analysis, which has not yet been sufficiently developed for this purpose."*
- **1778: Lagrange solved the Euler equations for two particular cases**
  - The case for steady, incompressible flow gave us the famous Bernoulli's equation
- **1849: The Navier-Stokes equations for viscous, compressible flows**
  - [boundary] conditions which must be satisfied at the surface of a solid in contact with the fluid…are unknown
- **1883: Reynolds characterized viscous flows:** *"…steady direct motion in round tubes is stable or unstable according as $\rho D U_m / \mu$ <1900 or >2000,…"*
- **1895: The Reynolds-averaged Navier-Stokes (RANS) equations for viscous, compressible, turbulent flow (*mean-mean* and *relative-mean* motions)**
  - For RANS equations to be usable, need to address the Closure Problem: express Reynolds stresses in terms of known or calculable quantities—turbulence modeling
  - For more than 100 years, quest for 'better' turbulence models has been the *"holy grail"*
- **Throughout the 1800s: Impressive advances in *Ideal-Fluid Dynamics* [*rotational (w/ vortex filaments) and irrotational (no vorticity) flows of ideal fluids (inviscid, incompressible)*]—fueled by advances in mathematics**

Topics

At the Dawn of the 20th Century…

- **17 December 1903 to be precise—the first manned, controlled, powered flight by the Wright brothers!**

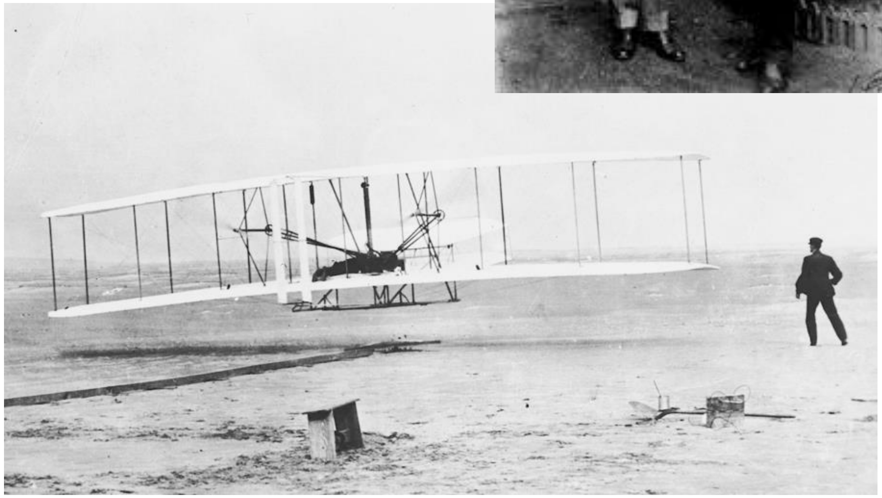
Orville Wright's telegram to his father:

Success. Four flights Thursday morning. All against twenty one mile wind. Started from level with engine power alone. Average speed through air thirty one miles. Longest 57 seconds. Inform press. Home Christmas.

Orville and Wilbur

"This flight lasted only twelve seconds, but it was nevertheless the first in the history of the world in which a machine carrying a man had raised itself by its own power into the air in full flight, had sailed forward without reduction of speed and had finally landed at a point as high as that from which it started.

- Orville Wright



- **Dramatic evolution of civil and military aviation followed**

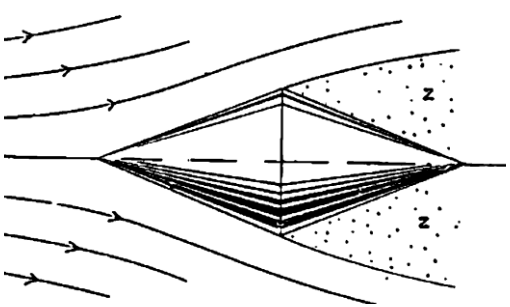
…12 Seconds Changed Human History Forever!

Source: Ref. 4.1 (p 87) and Internet

Analytical Fluid Dynamics

State of the Art at the Dawn of the 20th Century

- **AFD witnessed notable advances over preceding 150 years (1750–1900)**
  - Development of the governing equations of inviscid (Euler) and viscous flows (Navier-Stokes & RANS)
  - Advances in novel mathematical tools and techniques (such artifacts as sources, sinks, doublets, vortex filaments, etc.) used to obtain *analytical solutions* of irrotational (potential) and rotational flows of perfect or ideal fluids
- **But available AFD capabilities woefully inadequate to meet the emerging need of airplane engineering design**
- **AFD offered no satisfactory solution for the problem of resistance—a key need for airplane design!**
  - ***d'Alembert's paradox (1749-1752) remains unresolved even after 150 years!***
    - *"In a velocity field that is uniform at infinity and tangent to the body along its surface… [body] would suffer no force from the fluid, which is contrary to experience"*
  - **"Surface of Discontinuity" Theory proposed by Hermann von Helmholtz *(1858-1868)***
    - *"Any geometrically complete sharply-defined edge at which fluids flow past must tear itself from the most typical velocity of the remaining fluid and define a separation surface."*
    - Whole resistance being then due to the excess pressure region in front of the body, the dead-water or wake being at approximately the hydrostatic pressure of the fluid.



Source: Refs. 3.7, 4.1, 4.2

Analytical Fluid Dynamics

The Problem of Resistance Challenged Even the Brightest Minds!

On the Resistance of Fluids **(Lord Rayleigh F.R.S.)**

The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science, 2:13, 430-441, 1876

(Nearly 125 years after d'Alembert's Paradox was published!)

"There is no part of hydrodynamics more perplexing to the student than which treats the resistance of fluids. According to one school of writers a body exposed to a stream of perfect fluid would experience no resultant force at all, any augmentation of pressure on its face due to the stream being compensated by equal and opposite pressures on its rear…On the other hand it is well known that in practice an obstacle does experience a force tending to carry it downstream and of magnitude too great to be the direct effect of friction; while in many of the treatises calculations of resistance are given leading to results depending on the inertia of the fluid without any reference to friction."

John William Strutt
3rd Baron Rayleigh

Nobel Prize in Physics (1904)
12 Nov 1842 – 30 Jun 1919

Prevailing Wisdom:
Fluid Friction Too Small to Produce Significant Resistance Force!

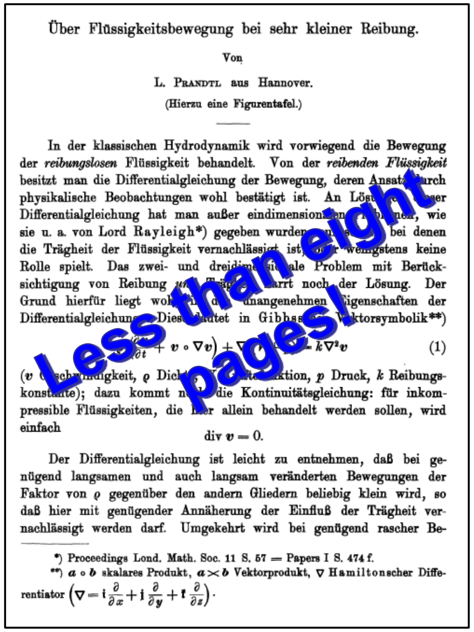
Source: Ref. 4.4; Wikipedia

Finally a Breakthrough in 1904!

Prandtl's Boundary Layer Theory

Über Flussigkeitsbeweging bei sehr kleiner Reibung.
Verhandlungen Des Dritten Internationalen Mathematiker-Kongresses, Heidelberg, Vom 8, Bis 13, August 1904, pp 484-491

*"The most important aspect of the problem is the behavior of the fluid on the surface of the solid body. The physical processes in the boundary layer [Grenzschicht] between fluid and solid body can be calculated in a sufficiently satisfactory way if **it is assumed that the fluid adheres to the walls, so that the total velocity is either zero or equal to the velocity of the body**. If, however, the viscosity is very small and the path of the fluid along the wall not too long, the velocity will have its normal value very near to the wall. **In the thin transition layer (Ubergangsschicht) the sharp changes of velocity, in spite of the small viscosity coefficient, produce noticeable effects.**"*



Über Flüssigkeitsbewegung bei sehr kleiner Reibung.

Von

L. Prandtl aus Hannover.

(Hierzu eine Figurentafel.)

In der klassischen Hydrodynamik wird vorwiegend die Bewegung der *reibungslosen* Flüssigkeit behandelt. Von der *reibenden Flüssigkeit* besitzt man die Differentialgleichung der Bewegung, deren Ansatz [illegible]urch physikalische Beobachtungen wohl bestätigt ist. An Lös[illegible] [illegible]ser Differentialgleichung hat man außer eindimensiona[illegible] [illegible], wie sie u. a. von Lord Rayleigh*) gegeben wurden, [illegible] bei denen die Trägheit der Flüssigkeit vernachlässigt ist, [illegible] wenigstens keine Rolle spielt. Das zwei- und dreidim[illegible]ale Problem mit Berücksichtigung von Reibung [illegible] [illegible]arrt noch der Lösung. Der Grund hierfür liegt wo[illegible] [illegible] unangenehmen Eigenschaften der Differentialgleichung[illegible] Dies[illegible]lautet in Gibbs[illegible] Vektorsymbolik**)

[illegible] $\left(\frac{\partial v}{\partial t} + v \circ \nabla v\right) + \nabla$ [illegible] $= k\nabla^2 v$ (1)

(v [illegible]schwindigkeit, ϱ Dich[illegible], [illegible]tion, p Druck, k Reibungskonst[illegible]te); dazu kommt n[illegible] die Kontinuitätsgleichung: für inkompressible Flüssigkeiten, die [illegible]er allein behandelt werden sollen, wird einfach

$$\operatorname{div} v = 0.$$

Der Differentialgleichung ist leicht zu entnehmen, daß bei genügend langsamen und auch langsam veränderten Bewegungen der Faktor von ϱ gegenüber den andern Gliedern beliebig klein wird, so daß hier mit genügender Annäherung der Einfluß der Trägheit vernachlässigt werden darf. Umgekehrt wird bei genügend rascher Be-

*) Proceedings Lond. Math. Soc. 11 S. 57 = Papers I S. 474 f.

**) $a \circ b$ skalares Produkt, $a \times b$ Vektorprodukt, ∇ Hamiltonscher Differentiator $\left(\nabla = \mathfrak{i}\frac{\partial}{\partial x} + \mathfrak{j}\frac{\partial}{\partial y} + \mathfrak{k}\frac{\partial}{\partial z}\right)$.

Less than eight pages!

2D BL equations

$$\varrho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) + \frac{dp}{dx} = k\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

Ludwig Prandtl

German Physicist
4 Feb 1875 – 15 Aug 1953

2D BL velocity profile

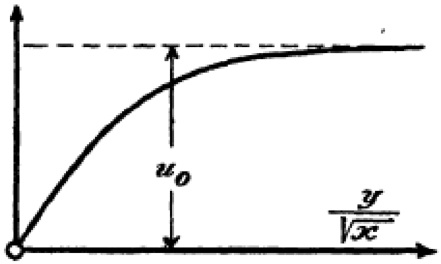


Fig. 1.

"A Most Extraordinary Paper of the 20th Century, and Probably of Many Centuries!" — Sydney Goldstein, Harvard Univ.

Source: Ref. 4.5 & 4.6, and Wikipedia

Resistance Formula for Thin Flat Plate!

Prandtl's Solution of Boundary Layer Equations

Über Flussigkeitsbeweging bei sehr kleiner Reibung.
Verhandlungen Des Dritten Internationalen Mathematiker-Kongresses, Heidelberg, Vom 8, Bis 13, August 1904, pp 484-491

Ludwig Prandtl

German Physicist
4 Feb 1875 – 15 Aug 1953

"If, as usual, d*p*/d*x* is given throughout, and furthermore the variation of u for the initial cross-section of the flow, then every problem of this kind may be mastered numerically, in that one can obtain from every value of *u* the corresponding $\partial u/\partial x$ by quadrature. With this and the help of one of the familiar approximate methods, one can repeatedly move a step at a time in the x direction. Of course a difficulty exists with various singularities arising at solid boundaries. The simplest case of the flow situations considered here is the one in which water flows along a thin flat plate. A reduction in the variables is possible here; one can put $u = f\left(\frac{y}{\sqrt{x}}\right)$. One comes up with a formula for the flow resistance using a numerical result of the resulting [ordinary] differential equation

$$R = 1.1 \cdots b\sqrt{k\rho l u_0^3}$$

(*b* width, *l* length of the plate, u_0 the velocity of the undisturbed water opposite the plate)."

- The corresponding skin-friction drag coefficient (for both surfaces of the plate) is

$$C_F = 2.2/\sqrt{Re} \text{ where } Re = \frac{(\rho\, u_0\, l)}{k}$$

- More accurate calculations later corrected the factor 2.2 to 2.656

A Remarkable Achievement!

Source: Ref. 4.5 & 4.6, and Wikipedia

Boundary Layer Separation and Vortex Generation

Über Flussigkeitsbeweging bei sehr kleiner Reibung.
Verhandlungen Des Dritten Internationalen Mathematiker-Kongresses, Heidelberg, Vom 8, Bis 13, August 1904, pp 484-491

"The most important result of the investigation for application is that, in certain cases, the flow will separate from the wall at a place completely determined by the external conditions. A fluid layer, which has been set in rotation by the friction at the wall, makes its way into the free fluid where, causing a complete transformation in the motion, it plays the same role as the Helmholtz surface of discontinuity."

Ludwig Prandtl

German Physicist
4 Feb 1875 – 15 Aug 1953

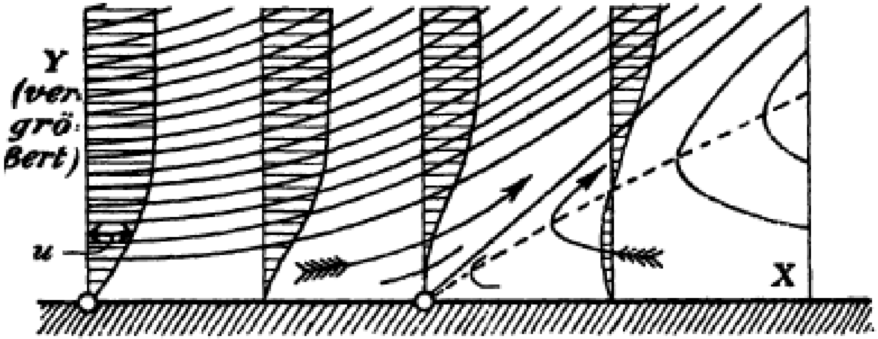


Fig. 2.

Necessary condition for flow separation:
pressure increase along the surface in the flow direction

"A change in the viscosity coefficient k alters the thickness of the vortex layer (proportional to $\sqrt{kl/\rho u}$) but everything else remains unchanged. Therefore, one can go over to the limit k = 0 and obtain the same flow picture."

A Singular Contribution of Enormous Lasting Influence for Explaining Otherwise Baffling Fluid Flow Phenomena

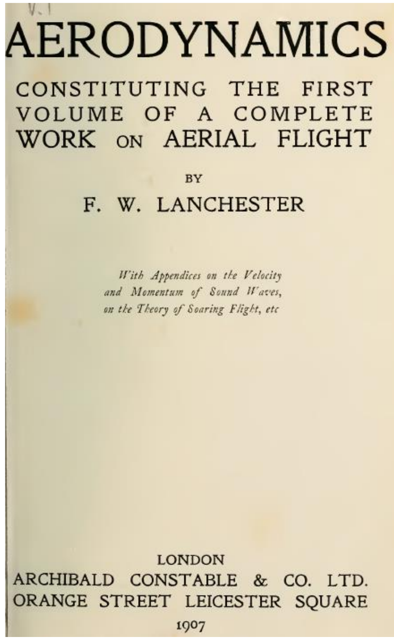
Source: Ref. 4.5 & 4.6, and Wikipedia

Aerodynamics: *State of the Art (1907)*

F. W. Lanchester

British Engineer
23 Oct 1868 - 8 Mar1946

"Numerical work has been done by the aid of an ordinary 25 cm. slide rule, with a liability to error of about 1/5th of 1 percent, an amount which is quite unimportant."



AERODYNAMICS

CONSTITUTING THE FIRST VOLUME OF A COMPLETE WORK ON AERIAL FLIGHT

BY

F. W. LANCHESTER

With Appendices on the Velocity and Momentum of Sound Waves, on the Theory of Soaring Flight, etc

LONDON
ARCHIBALD CONSTABLE & CO. LTD.
ORANGE STREET LEICESTER SQUARE
1907

"…the author desires to record his conviction that the time is near when the study of Aerial Flight will take its place as one of the foremost of the applied sciences, one of which the underlying principles furnish some of the most beautiful and fascinating problems in the whole domain of practical dynamics."

"In order that real and consistent progress should be made in Aerodynamics and Aerodonetics, apart from their application in the engineering problem of mechanical flight, it is desirable, if not essential, that provision should be made for the special and systematic study of these subjects in one or more of our great Universities, provision in the form of an adequate endowment with proper scope for its employment under an effective and enlightened administration."

"…the country in which facilities are given for the proper theoretical and experimental study of flight will inevitably find itself in the best position to take the lead in its application and practical development."

In Early 1900s, Aerodynamics Became a Most Exciting Research Frontier!

The First Half of the 20th Century: Golden Age of Analytical Aerodynamics

Source: Ref. 4.3, and Wikipedia

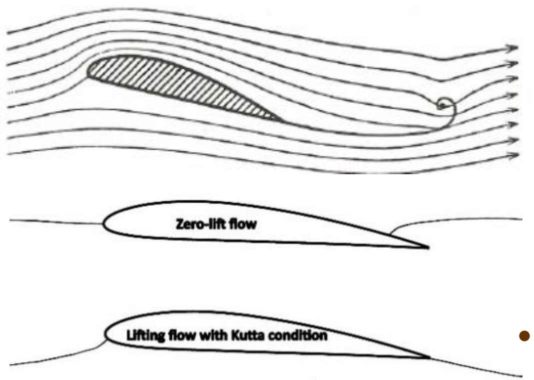
Analytical Aerodynamics: *the 1900s*

A Small Sampling of Pioneering Research

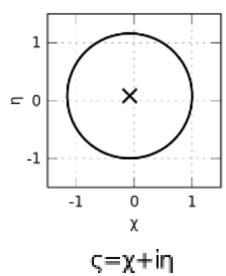
- **Kutta (1902)** – solution of inviscid 2D flow about circular-arc body at zero incidence with circulation and finite velocity at trailing edge

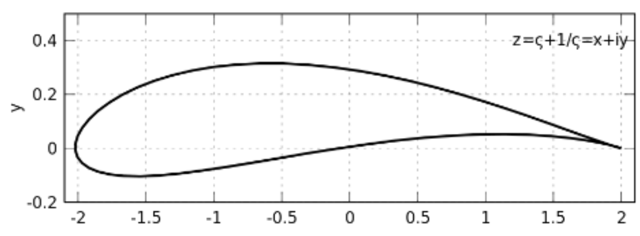
Martin Kutta

German Mathematician
3 Nov 1867 – 25 Dec 1944



- **Prandtl-Meyer (1908)** – oblique shocks and expansion fans in supersonic flows
- **Zhukovskii (1910)** design of airfoil sections using graphical construction



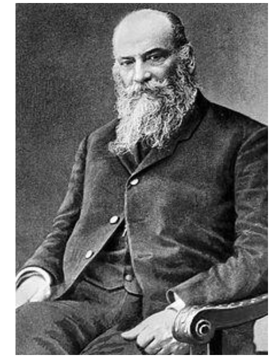


- **Prandtl (1904)** – boundary layer theory and vortex generation
- **Zhukovskii (1906)** – circulation theory of lift on 2D airfoils

$$l = \rho \Gamma V$$

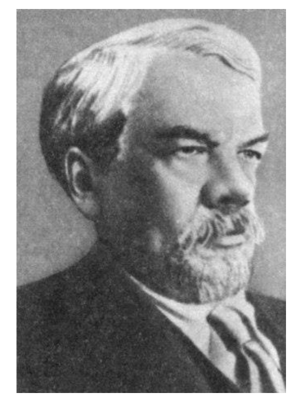
- **Chaplygin (1910) Postulate:** *"out of infinite number of theoretically possible solutions past an airfoil with sharp trailing edge, the flow that's nearest to experiment is the one with finite velocity at the trailing edge"*

Nikolay Zhukovsky



Russian Scientist, Mathematician
5 Jan 1847 – 17 Mar 1921

Sergey Chaplygin



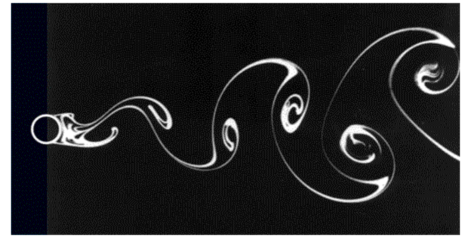
Russian Physicist, Mathematician, Engineer
5 Apr 1867 – 8 Oct 1942

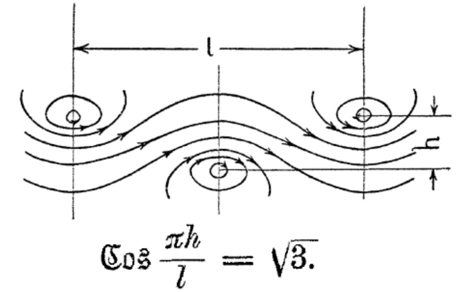
Source: Wikipedia; Refs. 4.6 to 4.9

Analytical Aerodynamics: *the 1910s*

A Small Sampling of Pioneering Research

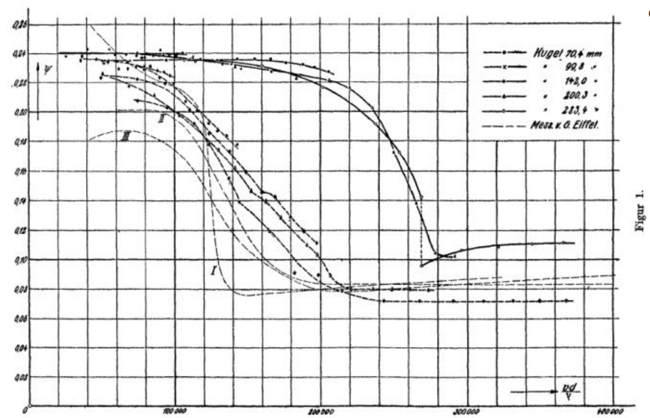
- **Kármán (1911)** – first paper on vortex street in the wake of 2D cylinders; referred to Boundary Layer theory to explain vortex formation

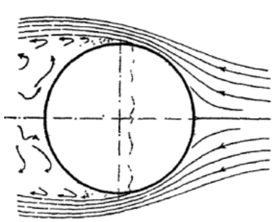




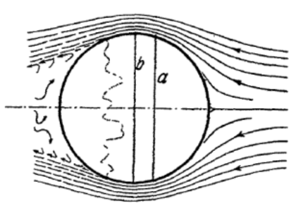
- **Blasius (1912)** – friction factor in turbulent pipe flows varied as inverse of the 1/4th power of Reynolds number, and velocity as the 1/7th power of the distance from the wall

- **Prandtl (1914)** – explained small drag coefficients for spheres with turbulent boundary layer that were first demonstrated by Eiffel in 1912





Figur 3.



Figur 4.

- **Prandtl (1918-1919)** – classic papers on 3D airfoil (wing) theory of large but finite aspect ratio

$$W = \varrho \int_a^b \Gamma w \, dx$$

$$w(x) = \frac{1}{4\pi} \int_a^b \frac{d\Gamma}{dx'} \cdot \frac{dx'}{x - x'}$$

- **Munk (1918)** – the term "induced drag" and the now well-known "Munk's stagger theorem"

- **Betz (1919) –** screw propeller with minimum energy loss

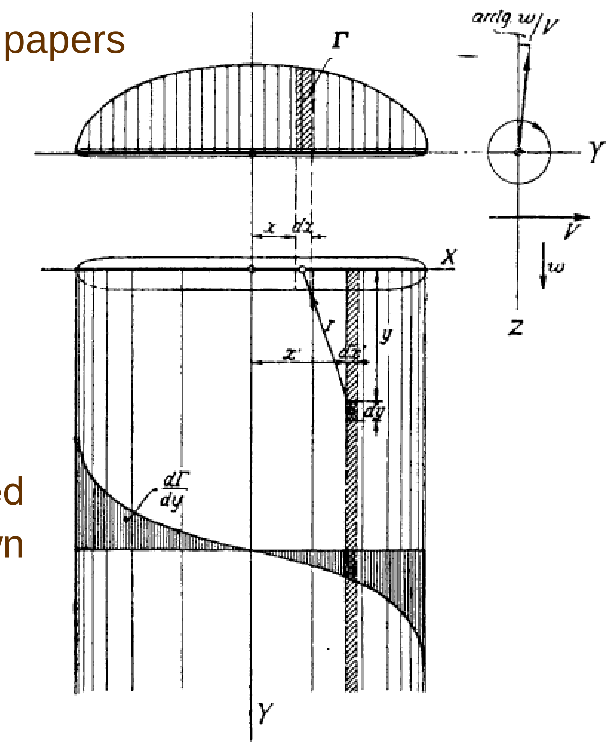


Fig. 4 [1]).

Source: Wikipedia; Refs. 4.6, 4.10 to 4.13

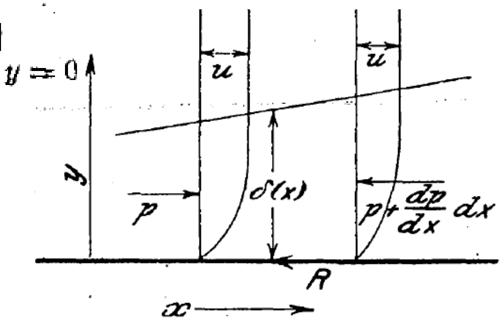
Analytical Aerodynamics: *the 1920s*

A Small Sampling of Pioneering Research

- **Trefftz (1921)** – estimate induced drag from wake integral in a far downstream "Trefftz plane"
- **Kármán (1921)** – momentum equations of boundary layer, and Kármán-Pohlhausen approximate method of integration

$$\frac{\partial}{\partial t}\int_0^\delta \rho u\,dy + \frac{\partial}{\partial x}\int_0^\delta \rho u^2 dy - u_0 \frac{\partial}{\partial x}\int_0^\delta \rho u\,dy = -\delta\frac{\partial p}{\partial x} - R$$

$$\tau_0 = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}$$



Flat plate skin friction formulas for laminar & turbulent boundary layers!

- **Taylor (1923)** – "Stability of viscous liquid contained between two rotating cylinders"

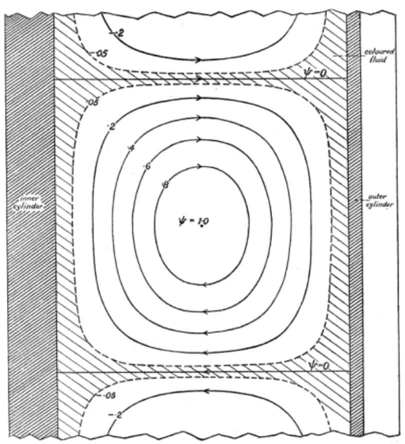


Fig. 5. Stream lines of motion after instability has set in, ψ positive.

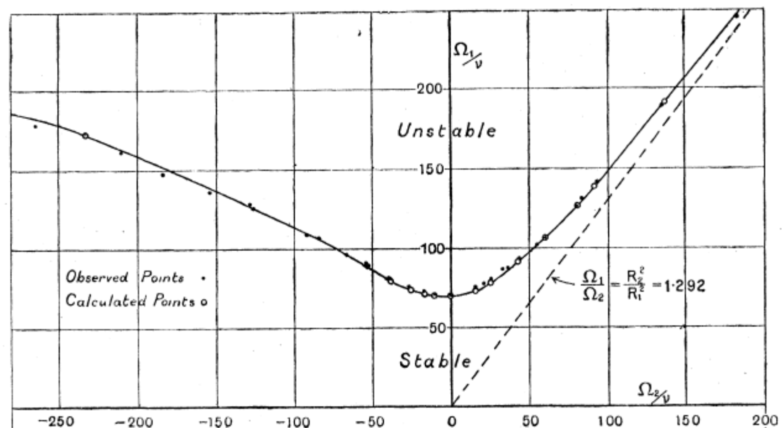


Fig. 18. Comparison between observed and calculated speeds at which instability first appears; case when $R_1 = 3{\cdot}55$ cm., $R_2 = 4{\cdot}035$ cm.

- **Prandtl (1925)** – "mixing path (or distance) theory" for turbulent flows with the proposition: *momentum is a transferable property*

$$\tau = \varrho l^2 \left|\frac{du}{dy}\right| \cdot \frac{du}{dy} \qquad \mu_T = \varrho l^2 \left|\frac{du}{dy}\right|$$

"…a first rough approximation."

- **Glauert (1928) –** Prandtl-Glauert rule for inviscid compressible flows: $C_p = C_{p0}/\beta \quad \beta^2 = 1 - M_\infty^2$

Theodore von Kármán

Hungarian-American Mathematician, Physicist, Aerospace Engineer
11 May 1881 – 6 May 1963

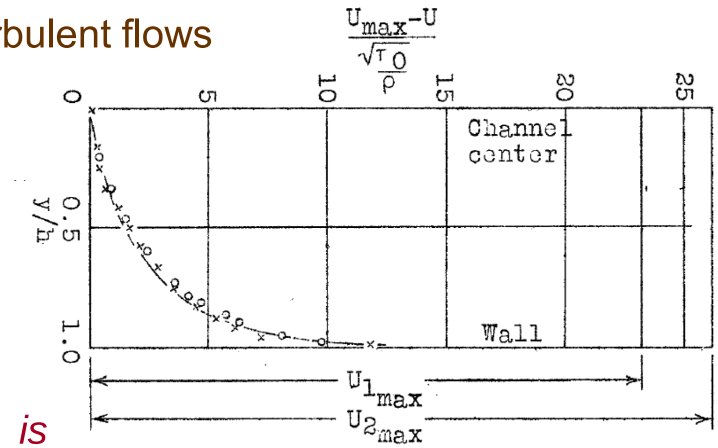
Source: Wikipedia; Refs. 4.6, 4.14 to 4.18

Analytical Aerodynamics: *the 1930s*

A Small Sampling of Pioneering Research

- **Kármán (1930)** – logarithmic "law of the wall" for planar turbulent flows

$$U_{max} - U = -\frac{1}{k}\sqrt{\frac{\tau_0}{\rho}}\left(\log\left(1 - \sqrt{\frac{y}{h}}\right) + \sqrt{\frac{y}{h}}\right)$$



  - U_{max} *is the difference between wall and channel center*
  - k *is a constant independent of dimensions and Reynolds number, appears to have a value 0.38*

- **Taylor (1932)** – Proposed that *vorticity, not momentum, is the transferable property* in his paper entitled "*The transport of vorticity and heat through fluids in turbulent motion*"

- **Taylor-Maccoll (1933)** – Derived and solved an ordinary differential equation (O.D.E.) with one unknown for supersonic flow past a cone

- **Taylor (1935)** – "Statistical theory of turbulence" – whole new direction to turbulent flow research!

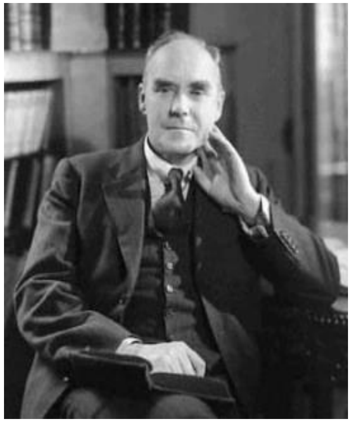
  <u>Predicted</u> Law of Decay of Turbulence behind grids and honeycombs

$$\frac{U}{u'} = \frac{5x}{A^2 M} + \text{constant.}$$

  A = a constant, *determined experimentally should be universal for all square grids;* M = *mesh length of a square mesh*

- **Taylor (1935-37)** – modified vorticity-transfer theory with application to flow in pipes

G.I. Taylor



British Physicist, Mathematician
7 Mar 1886 – 27 Jun 1975

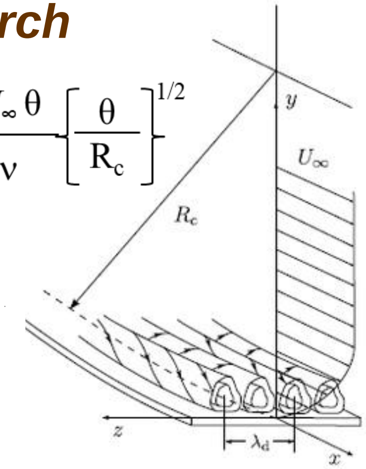
Source: Wikipedia; Refs. 4.6, 4.19 & 4.20

Analytical Aerodynamics: *the 1940s*

A Small Sampling of Pioneering Research

- **Görtler (1940)** – theoretical study of the instability of boundary layer flows on concave surfaces; instability occurs when **Görtler number**, G > 0.3

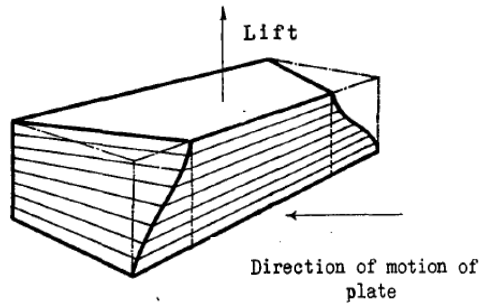
$$G = \frac{U_\infty \theta}{\nu}\left[\frac{\theta}{R_c}\right]^{1/2}$$



- **Busemann (1942-43)** – conical supersonic flow theory

Adolf Busemann

German Aerospace Engineer
20 Apr 1901 – 3 Nov 1986



Pressure distribution on a flat plate

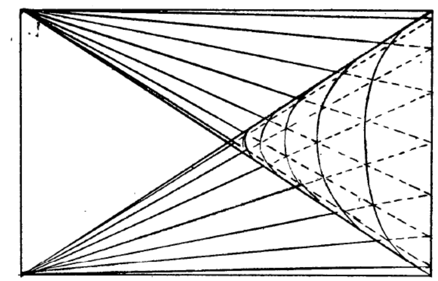


Figure 12. Superposition of edge influences for the rectangular plate at supersonic velocities

- **Jones (1946)** –theory of pointed wings (delta wings) of very small aspect ratio

$$C_L = \frac{\pi}{2} A\alpha$$

$$C_{D_i} = C_L \frac{\alpha}{2}$$

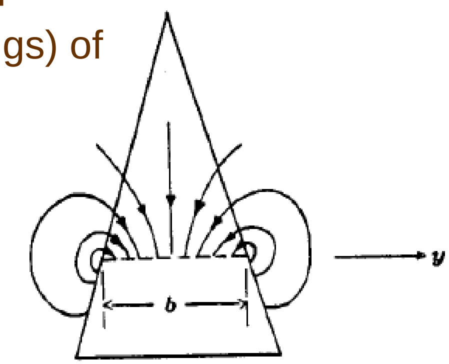


FIGURE 1.—Flow pattern.

$$c_l = \pi\alpha \frac{db}{dx}$$

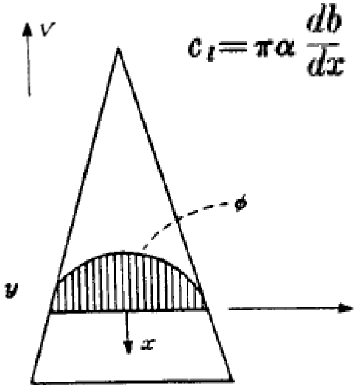


FIGURE 2.—Potential.

- **Tsien (1946) –** similarity laws of hypersonic flows

$$K = M_\infty (\delta/b)$$

- **Kármán (1947)** – similarity law of transonic flows

$$K = (1 - M_\infty)/(\tau\Gamma)^{2/3} \qquad \Gamma = (\gamma+1)/2; \ \gamma = C_p/C_v$$

If a series of bodies of same thickness distribution but different thickness ratios (δ/b *or* τ) are placed in streams of different M_∞, then the *flow patterns are similar* as long they all have equal values of K

- **Lighthill (1947)** – hodograph transformation in transonic flows

Source: Wikipedia; Refs. 4.6, 4.21 to 4.25

Analytical Aerodynamics:
Summary Assessment of Capabilities

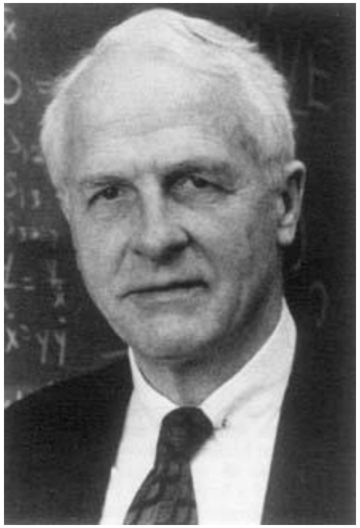
Author's Opinion

In spite of phenomenal advances in the first half of the 20th Century, analytical aerodynamics *(circa 1950)* remained inadequate for simulating realistic flows on *complex* geometries—*and remains so even today!*

"…no exact analytical model describing physically interesting flows that depend significantly on Re [Reynolds number] is known."

– Garrett Birkhoff, 1981

Garrett Birkhoff



American Mathematician
19 Jan 1911 – 22 Nov 1996

Source: Ref. 4.26, and Wikipedia

Value of Analytical Aerodynamics

In spite of severely limited capabilities of simulating realistic flows on complex geometries, it offers unique insights that other approaches do not!

"…**skillful application of the equations from the dynamics of ideal fluids quite often brings *clarity* into such phenomena** which in themselves are not independent of the viscosity. The vortex equations, in particular, proved themselves very useful. I may be allowed to mention the **vortex street** by which we are able to reproduce the mechanism of the form resistance with suitable approximation under stated conditions, although such a resistance is precluded in a fluid which is perfectly inviscid…Another striking example is the **theory of the induced drag of wings**, which likewise shows the extent of applying the vortex equations without overstepping the bounds of the dynamics of ideal fluids."

– *Theodore von Kármán, 1931*

Analytical Aerodynamics (a subset of AFD) Remains Indispensable for Better Understanding of Complex Flow Phenomena

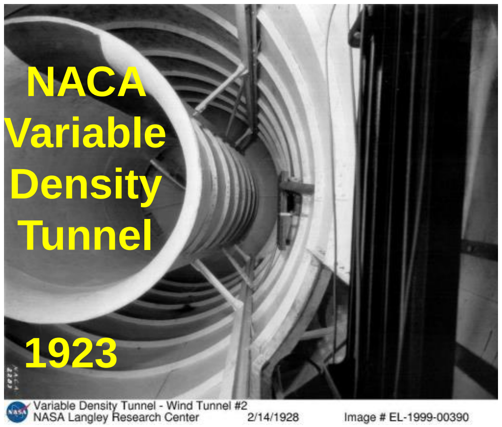
Source: Ref. 4.19

Experimental Aerodynamics: *1900 – 1950*

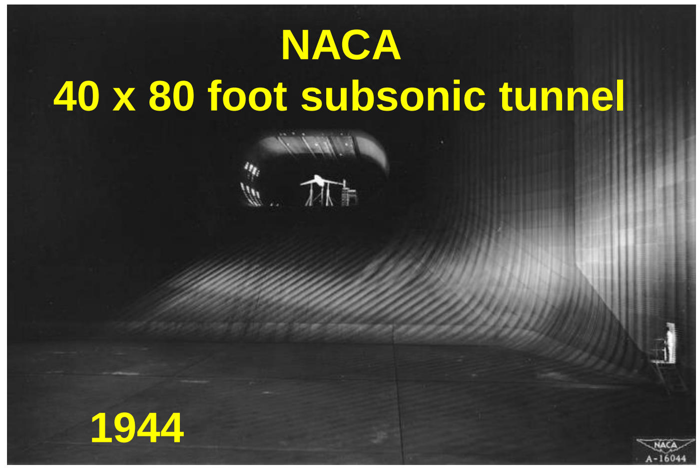
An Effective Means of Overcoming Inadequacies of AFD

Rapid advancements to support development of new airplane designs

- Bigger tunnels; high-speed tunnels; low-turbulence tunnels; special purpose tunnels; …



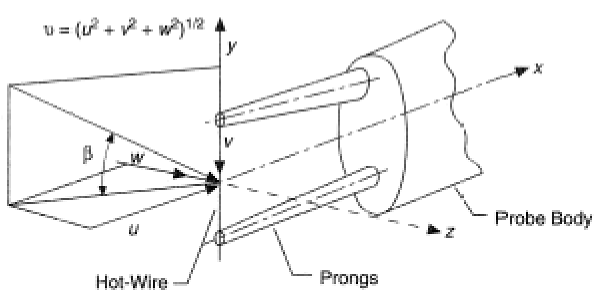
"data for 78 classical airfoil shapes: see TR 460, 1935"

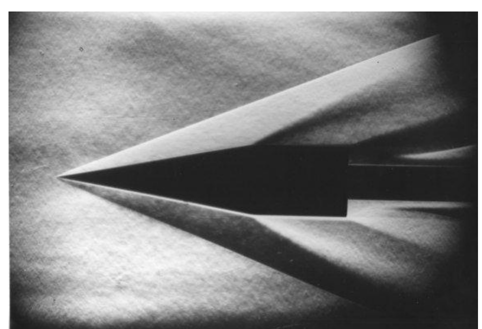


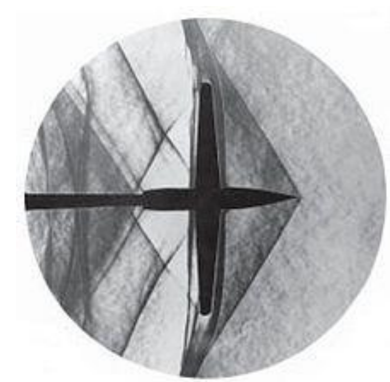
"aircraft development work"

"solve the mysteries of flight beyond Mach 1"

- Techniques and instruments for accurate measurements (e.g., hot-wire anemometry) and visualization (e.g., Schlieren, interferometry)







Source: NASA websites; Refs. 4.27 & 4.28

Genesis of Numerical Aerodynamics: *1910*

The Approximate Arithmetical Solution by Finite Differences of Physical Problems involving Differential Equations, with an Application to the Stresses in a Masonry Dam.

By L. F. RICHARDSON, *King's College, Cambridge.*

Read January 13, 1910

IX. *The Approximate Arithmetical Solution by Finite Differences of Physical Problems involving Differential Equations, with an Application to the Stresses in a Masonry Dam.*

By L. F. RICHARDSON, *King's College, Cambridge.*

Communicated by Dr. R. T. GLAZEBROOK, *F.R.S.*

Received (in revised form) November 2, 1909,—Read January 13, 1910.

§ 1. INTRODUCTION.—§ 1·0. The object of this paper is to develop methods whereby the differential equations of physics may be applied more freely than hitherto in the approximate form of difference equations to problems concerning irregular bodies.

Though very different in method, it is in purpose a continuation of a former paper by the author, on a "Freehand Graphic Way of Determining Stream Lines and Equipotentials" ('Phil. Mag.,' February, 1908; also 'Proc. Physical Soc.,' London, vol. xxi.). And all that was there said, as to the need for new methods, may be taken to apply here also. In brief, analytical methods are the foundation of the whole subject, and in practice they are the most accurate when they will work, but in the integration of partial equations, with reference to irregular-shaped boundaries, their field of application is very limited.

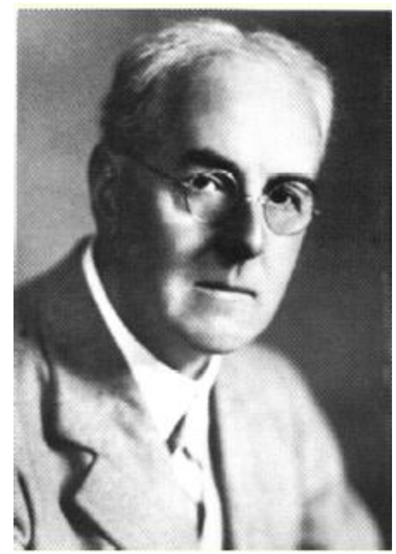
Both for engineering and for many of the less exact sciences, such as biology, there is a demand for rapid methods, easy to be understood and applicable to unusual equations and irregular bodies. If they can be accurate, so much the better; but 1 per cent. would suffice for many purposes. It is hoped that the methods put forward in this paper will help to supply this demand.

The equations considered in any detail are only a few of the commoner ones occurring in physical mathematics, namely:—LAPLACE'S equation $\nabla^2\phi = 0$; the oscillation equations $(\nabla^2+k^2)\phi = 0$ and $(\nabla^4-k^4)\phi = 0$; and the equation $\nabla^4\phi = 0$. But the methods employed are not limited to these equations.

The Number of Independent Variables.—In the examples treated in the paper this never exceeds two. The extension to three variables is, however, perfectly obvious. One has only to let the third variable be represented by the number of the page of a book of tracing paper. The operators are extended quite simply, and the same

VOL. CCX.—A 467. 2 R 2 24.5.10

Lewis Fry Richardson



FRS, British Mathematician, Physicist, Meteorologist, Psychologist

11 Oct 1881 – 30 Sep 1953

Source: Refs. 4.29 & 4.30

Richardson's Observations: *1910 Paper*

"The object of this paper is to ***develop methods*** *whereby the* ***differential equations*** *of physics* ***may be applied*** *more freely than hitherto* ***in the approximate form of difference equations to problems*** *concerning* ***irregular bodies****."*

*"…**analytical methods are the foundation** of the whole subject, and in practice they are the most accurate when they will work, but in the integration of partial equations, with reference to **irregular-shaped boundaries**, their **field of application is very limited**."*

"So far I have ***paid*** *piece rates* ***for the*** $\delta_x^2+\delta_y^2$ ***operation*** *of about* ***n/18 pence per co-ordinate point****, n being the number of digits. The chief trouble to the computers has been the intermixture of plus and minus signs. As to the rate of working,* ***one of the quickest boys averaged 2,000 operations*** $\delta_x^2+\delta_y^2$ *per week, for numbers of three digits,* ***those done wrong being discounted****."*

Extension to Fluid Flows

TO SIMULATE FLOW ABOUT IRREGULARLY SHAPED BODIES

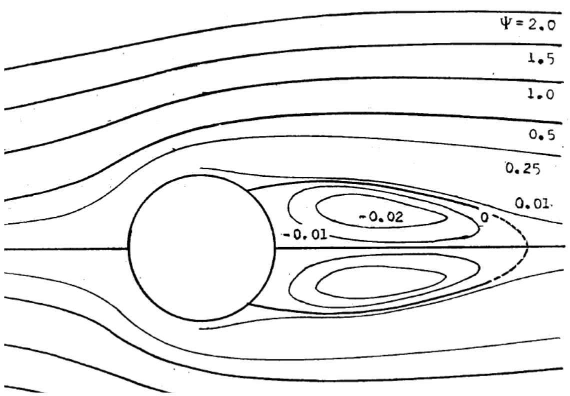
1. Use difference form of differential equations of *fluid flow* physics. **What**
2. Cannot apply analytical methods to irregularly shaped bodies. **Why**
3. Employ 'computers' [humans] to perform arithmetic operations. **How**

The What, the Why and the How of CFD (the rest is DETAIL!)

Source: Ref. 4.30

Numerical Aerodynamics: *1910 – 1950*

- **Pioneering Foundational Research in Numerical Methods Parallels Exciting Research in Analytical Aerodynamics**
  - **Richardson (1910)** – point iterative scheme for Laplace's equation
  - **Liebmann (1918)** – improved version of Richardson's method with faster convergence
  - **Courant, Friedrichs, and Lewy (1928)** – uniqueness and existence of numerical solutions of PDEs (*origins of the CFL condition well known to all "CFDers"*)
  - **Southwell (1940)** – improved relaxation scheme tailored for hand calculations
  - **Frankel (1950)** – first version of successive over-relaxation scheme for Laplace's equation
  - **O'Brien, Hyman, and Kaplan (1950)** – von Neumann method for evaluating stability of numerical methods for time-marching problems
- **Early Adopters**
  - **Thom (1929-1933)** – flow past circular cylinders at low speeds by numerically solving steady viscous flow equations: *stream function–vorticity (ψ–ζ) formulation of the N-S equations*
  - **Kawaguti (1953) –** flow past circular cylinder at Re = 40
    - 232 mesh points for half flow region
    - Iterative procedure is considered converged when difference between successive approximations for ψ and ζ does not exceed 0.3% of maximum value for the last 4 cycles
    - ***"The numerical integration in this study took <u>about one year and a half with twenty working hours every week</u>, with a considerable amount of labor and endurance."***



The Bottleneck: Slow & Laborious Computing

Source: Refs. 4.26 & 4.31-4.33

COLLEGE OF ENGINEERING
KEVIN T. CROFTON DEPARTMENT OF
AEROSPACE AND OCEAN ENGINEERING
VIRGINIA TECH.

A Vision for the Future (1946)

*"Our **present analytical methods seem unsuitable for the solution of** the important problems arising in connection with **non-linear partial differential equations**…The truth of this statement is particularly striking in the field of **fluid dynamics**."*

*"The **advance of analysis is,** at this moment, **stagnant** along the entire front of non-linear problems…Although the main mathematical difficulties have been known since the time of Riemann and of Reynolds, and although as brilliant a mathematical physicists as Rayleigh has spent a major part of his life's effort in combating them, yet **no decisive progress has been made** against them—indeed hardly any progress which could be rated as important…"*

*"…many branches of both pure and applied mathematics are in **great need of computing instruments to break the present stalemate** created by the failure of the purely analytical approach to nonlinear problems."*

John von Neumann

Hungarian-American Mathematician, Physicist, Computer Scientist
28 Dec 1903 – 8 Feb 1957
1999 Financial Times Person of the Century

"… really efficient high-speed [digital] computing devices may, in the field of non-linear partial differential equations as well as in many other fields…provide us with those heuristic hints which are needed in all parts of mathematics for genuine progress."

These are excerpts from the first paper in Ref. 4.35 entitled "ON THE PRINCIPLES OF LARGE SCALE COMPUTING MACHINES. *This paper was never published. It contains material given by von Neumann in a number of lectures, in particular one at a meeting on **May 5, 1946**, of the Mathematical Computing Advisory Panel, Office of Research and Inventions, Navy Department, Washington, D.C. The manuscript from which this paper was taken also contained material (not published here) which was published in the Report, "Planning and Coding of Problems for an Electronic Computing Instrument".*

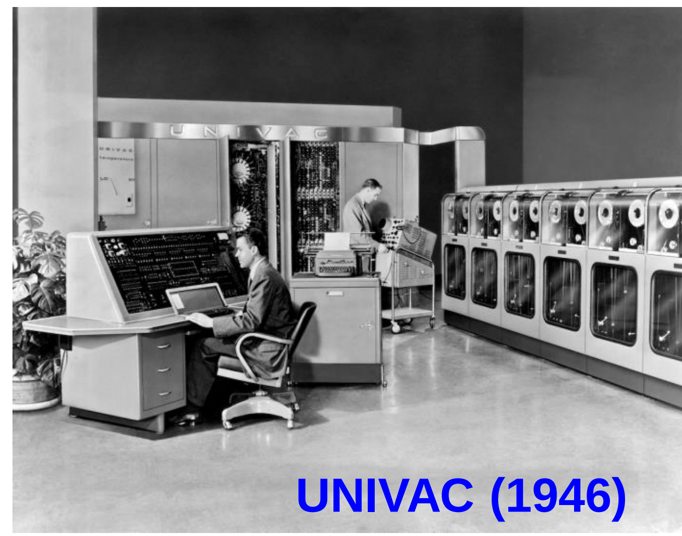
Note: Highlighting by the author.

Source: Refs. 4.34 & 4.35

Digital Computers: *1930 – 1950*

- **Alan Turing (1936)** – *a universal machine capable of computing anything that is computable*
- **Atanasoff (1937)** – *first computer without gears, cams, belts and shafts*
- **Atanasoff and Berry (1941)** – *a computer that can solve 29 equations simultaneously, and store information on its main memory*
- **Mauchly and Eckert (1943-44)** – *Electronic Numerical Integrator and Calculator* ***(ENIAC)*** *using 18,000 vacuum tubes*
  - ✓ Speed: 500 floating point operations per second
  - ✓ Size: 1,800 square feet
- **Mauchly and Presper (1946)** – *Universal Automatic Computer* ***(UNIVAC)****, the first commercial computer for business and government*

ENIAC (1943-44)



UNIVAC (1946)

The Key to Converting von Neumann's Vision into Reality!

Source: Ref. 4.36

Lecture 4: Overarching Takeaways

By 1950, all fundamental ingredients were in place for the evolution of an exciting new field of [what we call] Computational Fluid Dynamics (CFD).

In the second half of the 20th century, phenomenal advances in CFD methods and computing capabilities fueled the evolution of Applied Computational Aerodynamics (ACA).

ACA Evolution was Driven by the Promise of CFD Serving as a Powerful "Alternative" to AFD and EFD for Simulating Aerodynamics of Irregularly Shaped Bodies!

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