AOE 4144: Applied CFD

5. Evolution of Applied Computational Aerodynamics (1 of 5)

The 5th of 12 lectures by Prof. Raj to share his perspective on effective application of computational aerodynamics to aircraft design.

Each lecture contains excerpts from the presentation shown below describing his exciting journey on a long and winding road for more than five decades!

Reflections on the Effectiveness of Applied Computational Aerodynamics for Aircraft Design

https://www.aoe.vt.edu/people/emeritus/raj/personal-page/reflections-on-ACA-effectiveness.html

Pradeep Raj, Ph.D.
Collegiate Professor Emeritus
Kevin T. Crofton Department of Aerospace and Ocean Engineering
Virginia Tech, Blacksburg, Virginia, USA
http://www.aoe.vt.edu/people/emeritus/raj.html

Program Management Director, Lockheed Martin (Retired)
Deputy Director, Technology Development & Integration
The Skunk Works®, Palmdale, California, USA

Advanced Development Programs

LOCKHEED MARTIN

Lecture 4: Key Takeaways

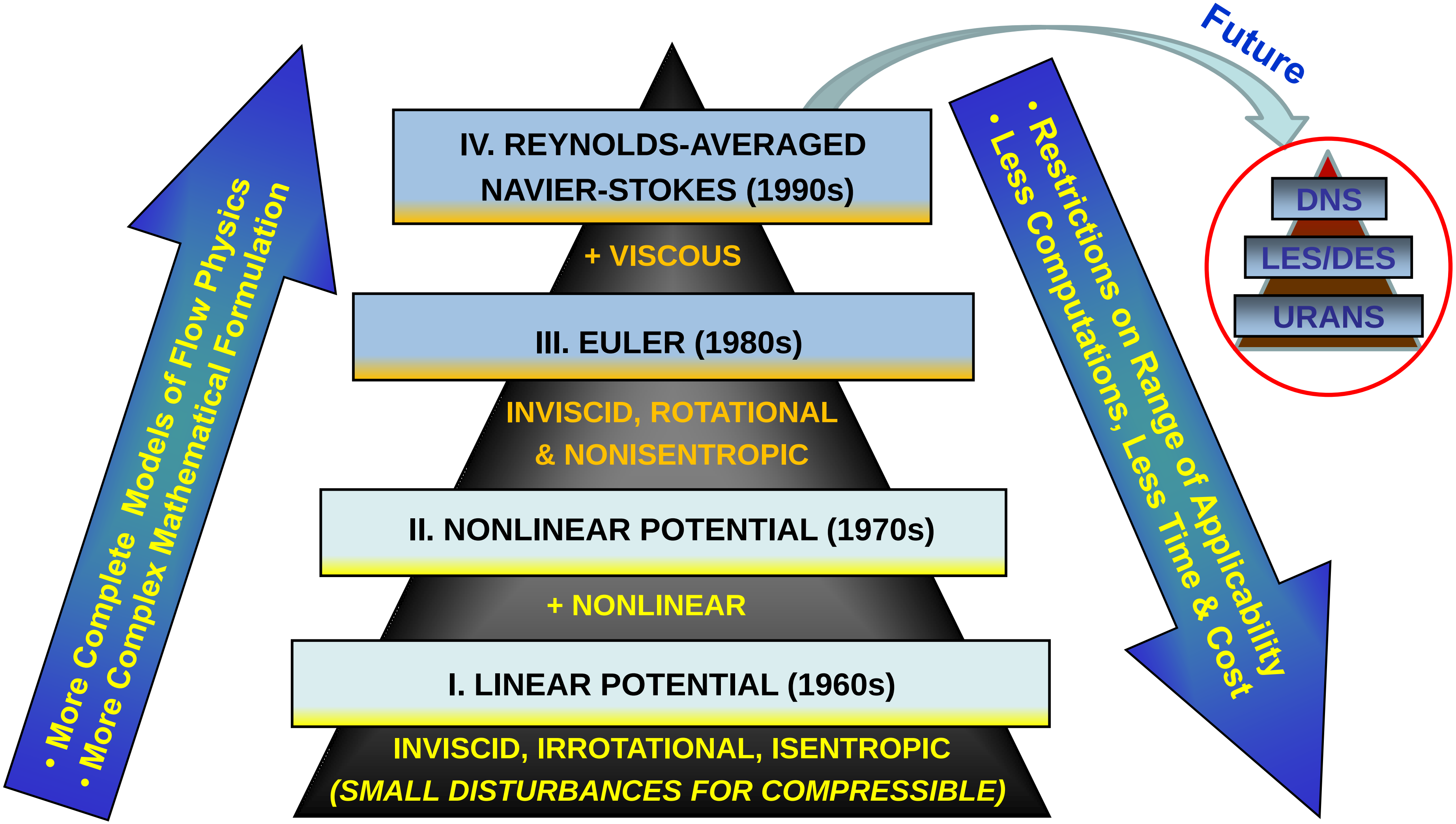
- **1903: the first manned, controlled, powered flight by the Wright brothers!**
- **Even after 150 years of noteworthy progress, Analytical Fluid Dynamics woefully inadequate to meet the emerging airplane design needs**
  - No solution of the problem of resistance in sight. *d'Alembert's paradox rules!*
- **1904: A breakthrough—Prandtl's Boundary Layer theory!**
  - *"A most extraordinary paper of the 20th century, and probably of many centuries!"*
- **The first 50 years of the 20th century (1900-1950) witnessed phenomenal advances in Analytical Aerodynamics, but…analytical models remained inadequate for simulating realistic flows on irregularly shaped bodies**
  - EFD provided the best means of solving practical engineering problem
- **1910: Richardson laid the foundation of Numerical Fluid Dynamics**
  - Use difference form of differential equations; employ human computers to perform resulting arithmetic operations; applicable to irregularly shaped bodies, but…
  - Human computers were the bottleneck!
- **1930 - 1950: Digital computers evolved**
  - Key to realizing von Neumann's 1946 vision: "*really efficient high-speed [digital] computing devices may break the present stalemate created by the failure of the purely analytical approach to nonlinear problems*"

By 1950, all basic ingredients were in place for the evolution of Computational Fluid Dynamics (CFD)

Topics

Evolution of ACA

Capabilities Directly Related to Four Levels of CFD Methods



- More Complete Models of Flow Physics
- More Complex Mathematical Formulation

IV. REYNOLDS-AVERAGED NAVIER-STOKES (1990s)

+ VISCOUS

III. EULER (1980s)

INVISCID, ROTATIONAL & NONISENTROPIC

II. NONLINEAR POTENTIAL (1970s)

+ NONLINEAR

I. LINEAR POTENTIAL (1960s)

INVISCID, IRROTATIONAL, ISENTROPIC
(SMALL DISTURBANCES FOR COMPRESSIBLE)

- Restrictions on Range of Applicability
- Less Computations, Less Time & Cost

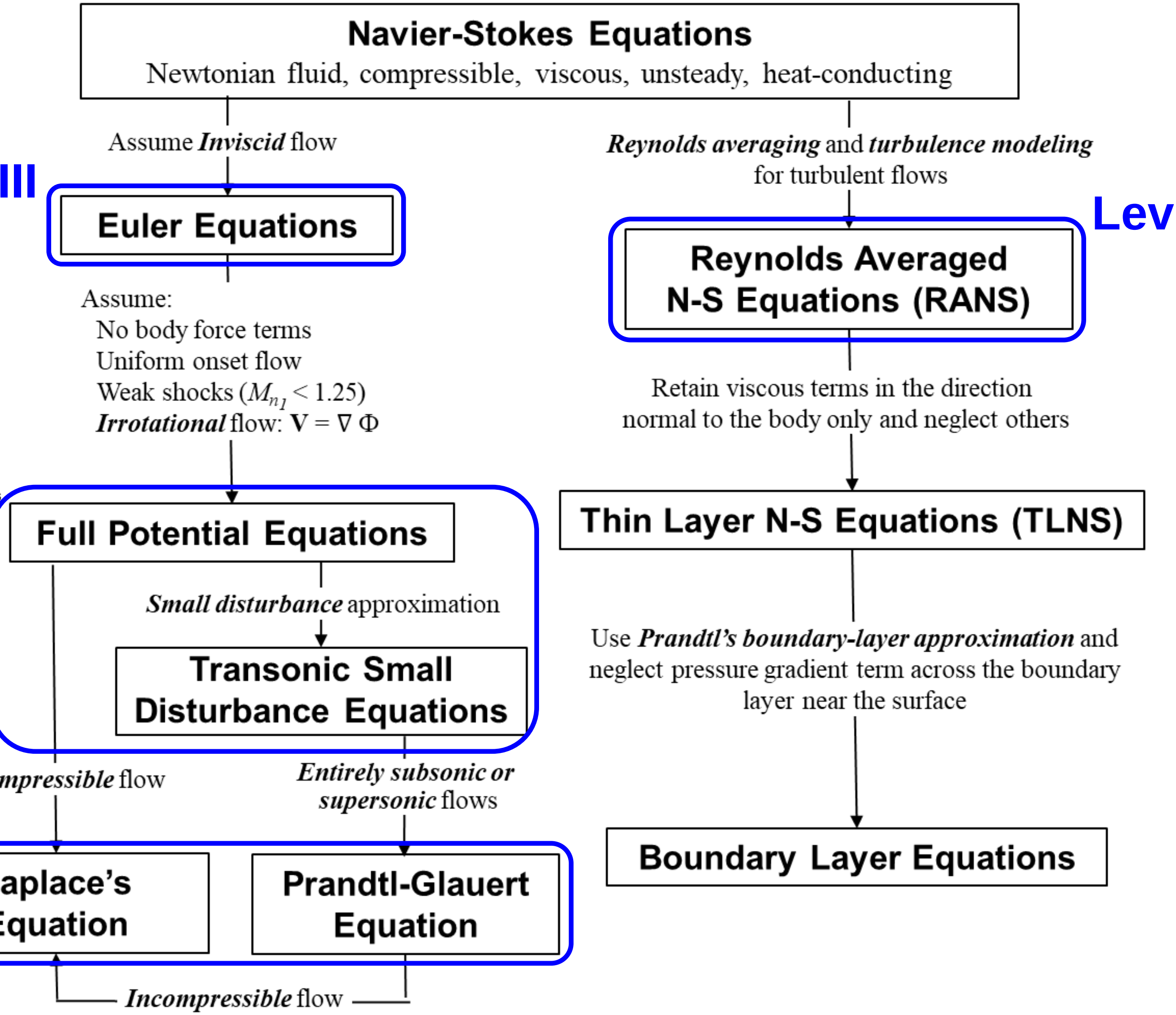
Future

DNS
LES/DES
URANS

Paced by Impressive Advances Since The 1950s

Note: Time frames in parenthesis indicate *widespread* adoption by industry

Approximations of N-S Equations Mapped to Four Levels of CFD



Navier-Stokes Equations
Newtonian fluid, compressible, viscous, unsteady, heat-conducting

Assume *Inviscid* flow

Level III

Euler Equations

Assume:
No body force terms
Uniform onset flow
Weak shocks ($M_{n_1} < 1.25$)
Irrotational flow: $\mathbf{V} = \nabla \Phi$

Reynolds averaging and *turbulence modeling* for turbulent flows

Level IV

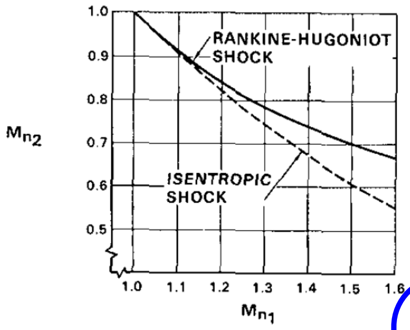
Reynolds Averaged N-S Equations (RANS)

Retain viscous terms in the direction normal to the body only and neglect others

Thin Layer N-S Equations (TLNS)

Use *Prandtl's boundary-layer approximation* and neglect pressure gradient term across the boundary layer near the surface

Boundary Layer Equations



Level II

Full Potential Equations

Small disturbance approximation

Transonic Small Disturbance Equations

Incompressible flow

Entirely subsonic or supersonic flows

Level I

Laplace's Equation

Prandtl-Glauert Equation

Incompressible flow

Adapted from Fig. 2-10, Configuration Aerodynamics by W.H. Mason

On “Fidelity” and “Credibility” of CFD Methods for Aerodynamic Simulations

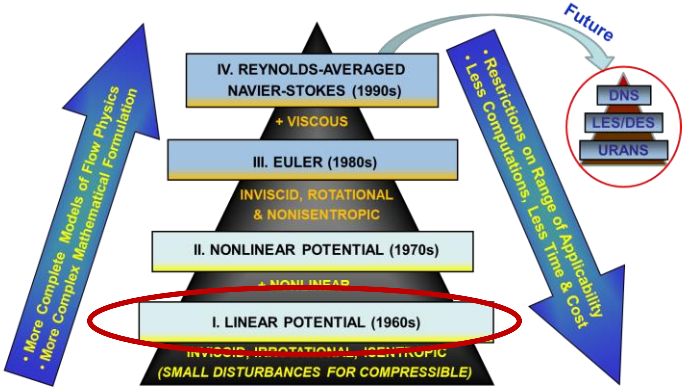
- Many CFD developers characterize *lower level* (potential [flow]) methods as **“*low fidelity*”** and **“*higher level*”** (Euler and RANS) methods as **“*high fidelity.*”** In this context, ***fidelity*** **implies *exactness*** with which governing equations approximate flow physics, not necessarily ***trustworthiness*** *of computational solutions in replicating reality.* ***For ACA, credibility implying trustworthiness of solutions is of paramount importance***.
- Experience has shown that ***higher level RANS methods do not necessarily produce credible data especially for complex flows*** that are dominated by vortices and boundary-layer separation. Therefore, one could argue that RANS methods are not *“high fidelity”* for replicating complex flows. When considering *fidelity,* more is <u>not</u> always better. **Using the “highest fidelity” CFD *in all instances* can lead to misuse of valuable resources.**
- Since each CFD method is (should be?) carefully designed to solve a selected set of equations as *accurately* as possible, a potential flow method may not be <u>inherently</u> *low fidelity*—as long as the method is *accurately* solving the governing potential flow equations, and *producing credible results for the target application*.
- **For ACA, it’s the *<u>credibility of aerodynamic data</u>* that is of utmost importance. Customers must have enough trust in data *<u>to use it in making decisions without incurring undue risk</u>*. This requires that data produced by a CFD method closely replicate reality. Validation is the most common approach for assessing credibility—albeit not without its own set of challenges to be highlighted later.**

What Matters Most to the Customer is Results, Not Tools!

Level I

Linear Potential Methods

1950s – present



Flow Model

- **Inviscid, Irrotational, Isentropic** ***(Small Disturbances for Compressible Flow)***

$$U = U_\infty + \nabla \phi$$

$$(\phi_{tt} + 2U_\infty \phi_{xt})/a_\infty^2 = (1 - M_\infty^2)\phi_{xx} + \phi_{yy} + \phi_{zz}$$

  - ✓ Linear second-order PDEs with appropriate boundary conditions
  - ✓ Laplace’s equation for steady, incompressible flow
  - ✓ Prandtl-Glauert equation for steady, compressible flow
  - ✓ *Wakes not captured as part of the solution—must be explicitly modeled*

Applicability

- Attached flows that are entirely subsonic or supersonic; *not transonic*
- Flows not dominated by shocks, vortices, or boundary-layer separation

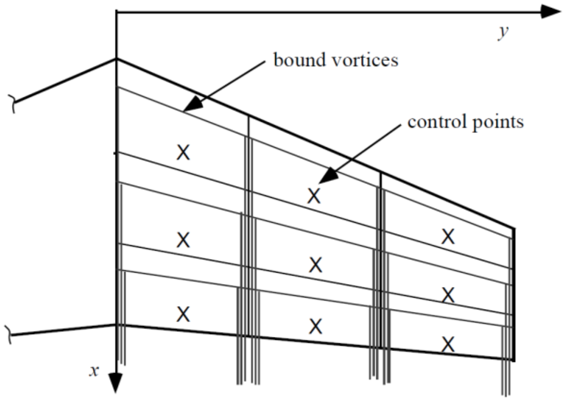
Refs. 5.1.1 to 5.1.45

Linear Potential Methods (LPMs)

Overview

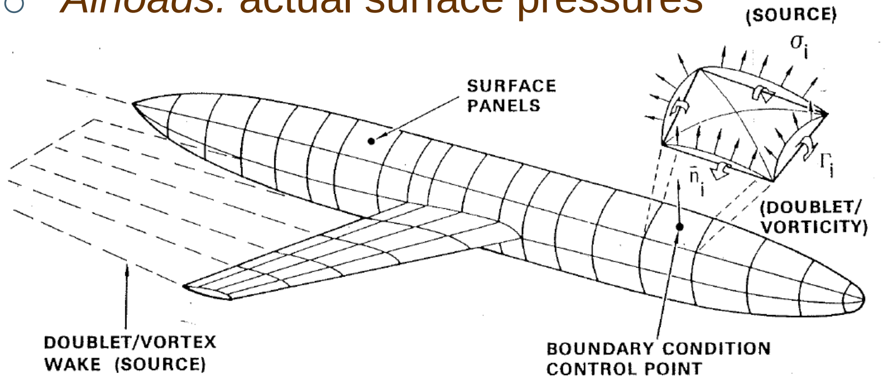
- **Basic Formulation**
  - Discretize geometry into small elements
  - Distribute singularities (source, doublets, vortex filaments) on each element
  - Impose no-normal-flow boundary condition (BC) at control points (one per element), and Kutta condition at sharp trailing edge
  - Solve resulting system of linear algebraic equations to determine singularity strengths
  - Use Bernoulli's equation to compute airloads

- **Vortex Lattice Methods (VLMs)**
  - *Geometry:* mean surface
  - *Singularity type:* horseshoe vortices
  - *BCs:* control points on mean surface
  - *Airloads:* net pressure



Trailing vortices extend to infinity

- **Panel Methods**
  - *Geometry:* actual surface
  - *Singularity type:* sources, doublets or both
  - *Singularity distribution:* constant, linear or higher order
  - *BCs:* control points on actual surface
  - *Airloads:* actual surface pressures

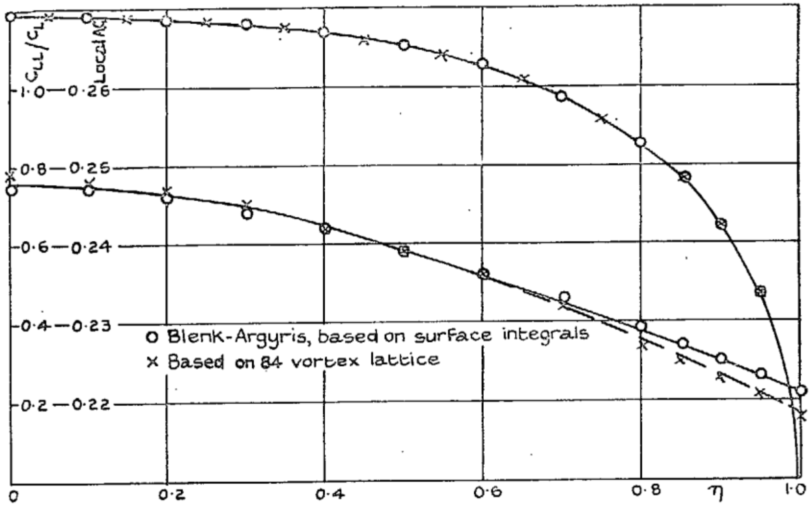


LPMs (VLMs & Panel Methods): Today's Workhorse!

Birth of Vortex Lattice Methods

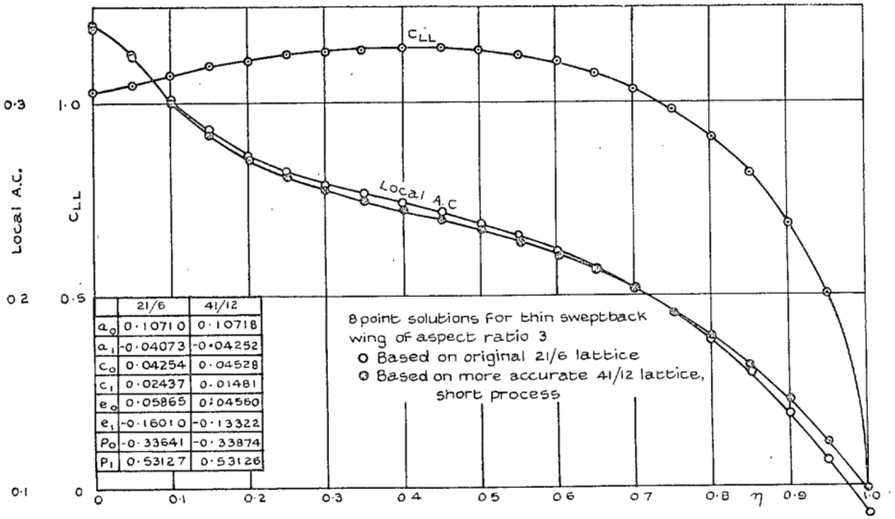
1940s

- **V.M. Falkner (1949): "The Scope and Accuracy of Vortex Lattice Theory"** **Report & Memoranda 2740, Aeronautical Research Council, United Kingdom**
  - Research motivated by the need to calculate loading distribution on a wing of arbitrary plan form including wing twist, discontinuities due to flaps, compressibility, etc.; initiated in early '40s
  - Paper outlines principles of using vortex lattice to solve potential flow problems in lifting plane theory; highlights key developments from Falkner's R&M 2591 (1947) and R&M 1910 (1943)



Thin Rectangular Wing: Λ = 0º, AR = 6

84 vortex lattice: 14 spanwise, 6 chordwise



	21/6	41/12
a_0	0·10710	0·10718
a_1	-0·04073	-0·04252
c_0	0·04254	0·04528
c_1	0·02437	0·01481
e_0	0·05865	0·04560
e_1	-0·16010	-0·13322
P_0	-0·33641	-0·33874
P_1	0·53127	0·53126

Thin Sweptback Wing: Λ = 45º, AR = 3

21 spanwise, 6 chordwise and 41 spanwise, 12 chordwise

- **Variations were tried extensively throughout industry during the 1950s**

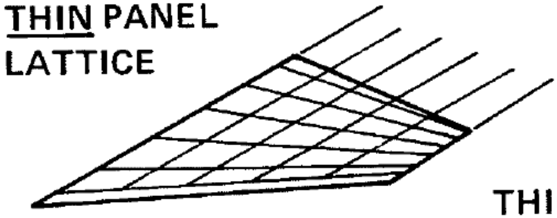
Advances in Electronic Computers and Numerical Methods in 1960s Made Practical VLM Applications Possible

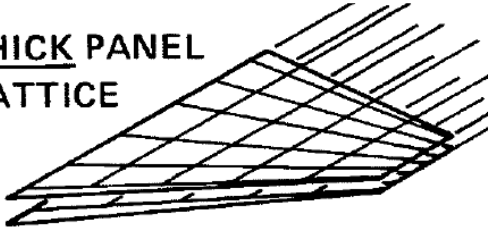
Source: Ref. 5.1.1

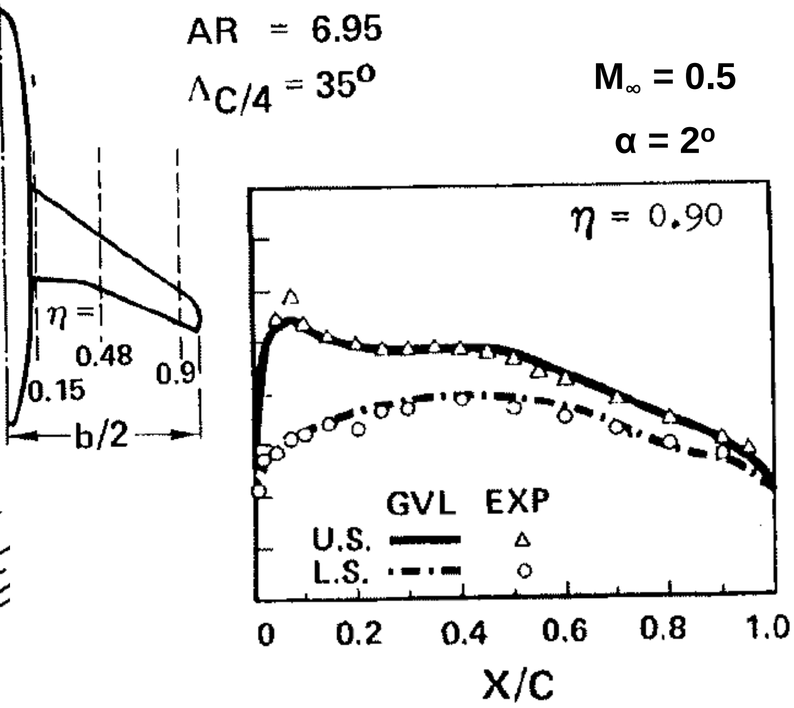
Vortex Lattice Methods (VLMs)

Rapid Development (1960s & 70s)

- **Rubbert (1964)**
    - Non-planar Vortex Lattice Methods; arbitrary wings—*Boeing Co. Document D6-9244*
- **Margason and Lamar (1971)**
    - Vortex-lattice Fortran program for estimating subsonic aerodynamic characteristics of complex planforms—*NASA TN D-6142*
- **Vortex-Lattice Utilization workshop (1976)**
    - Compilation of many papers—*NASA SP-405*
- **Miranda, Elliott and Baker (1977)**
    - A generalized vortex-lattice (GVL) method for subsonic and *supersonic* flow applications, the VORLAX code—*NASA CR 2865*







Falkner's Method Extended and Adapted to Electronic Computers

Source: Refs. 5.1.2 – 5.1.5

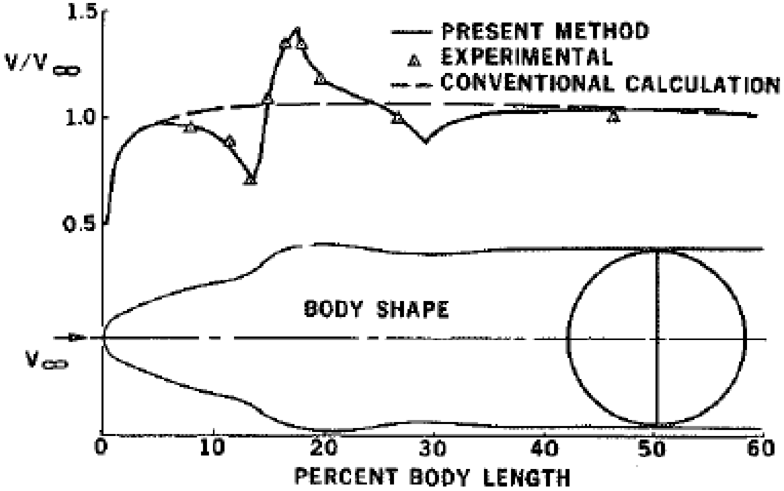
Birth of Panel Methods

1950s

- **A.M.O. Smith and J. Pierce, Douglas Aircraft Co., Long Beach, CA**
  - **Non-circulatory plane [2-D] and axially symmetric flows**
  - **1953**--Serious work began to solve **Neumann problem**
    - ***Continuous source distribution on surface panels***
  - **1954**--Programming on **IBM/701** *in machine language!*
  - Test cases selected based on availability of theoretical [analytical] solutions
  - From 24-point body of revolution solutions in 1954 to **150-points** by the end of **1955**!
  - DAC financed all work through **1958**
  - ONR contract: extend the method to 3-D non-lifting flows
- **DAC Report E.S. 26988, April 1958**

A.M.O. Smith

Chief Aerodynamics Engineer, Research
2 July 1911 – 1 May 1997



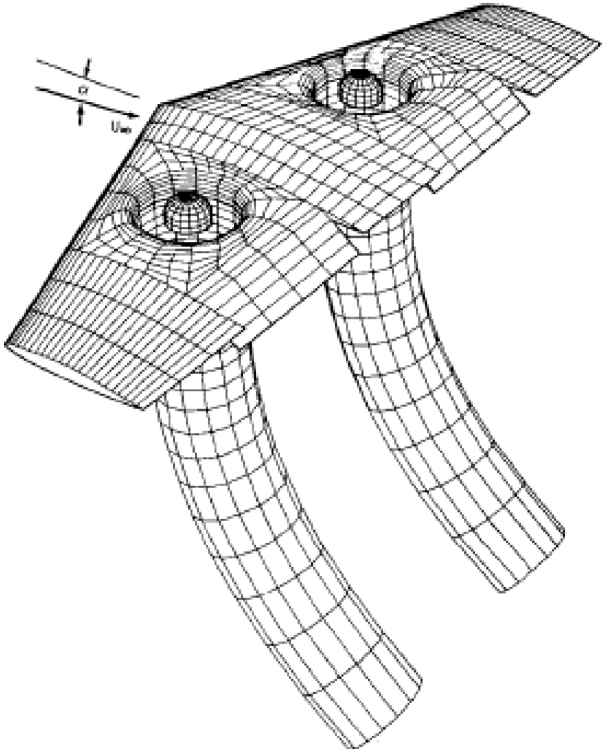
And the Rest is History!

Source: Refs. 5.1.6 & 5.1.7

Panel Methods

Rapid Development (1960s & 70s)

- **Hess (1962)**
  - Arbitrary bodies of revolution with axes perpendicular to the free stream direction—*Journal of the Aerospace Sciences*
- **Hess and Smith (1967)**
  - Extensive description of panel methods—*Progress in Aeronautical Sciences, Vol. 8* (138 pages!)
- **Rubbert and Saaris (1968)**
  - Incompressible flow; arbitrary configurations; source and doublet distributions—Fan-in-wing simulation, *SAE Paper 680304*
- **Hess (1970)**
  - Arbitrary 3-D lifting bodies—*McDonnell Douglas Rept. MDC J0971-01 (Also in Comp. Methods in Applied Mechanics and Engineering, 1974)*
- **Woodward (1973)**
  - Subsonic or *supersonic* flow; wing-body-tail configurations; source and vortex distributions—*NASA CR-2228*



Panels for a fan-in-wing configuration

Powerful Capability to Simulate Flow About Realistic Geometries to Support Aircraft Design Needs

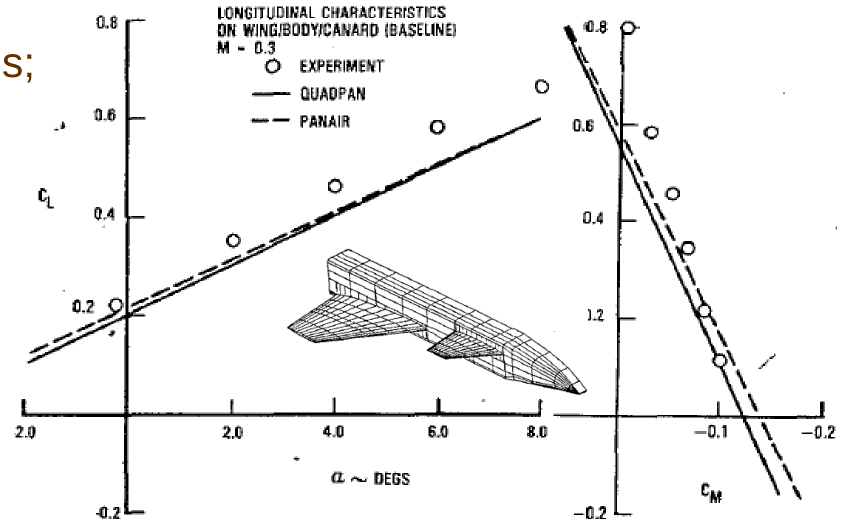
Source: Refs. 5.1.8 – 5.1.12

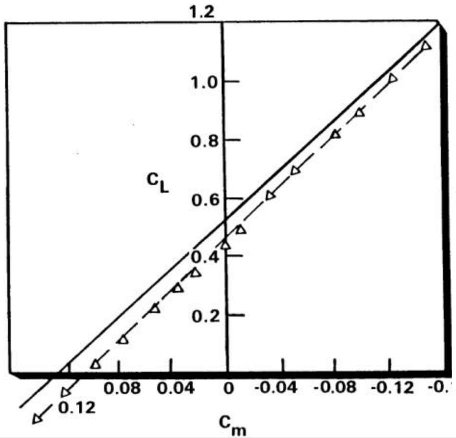
Panel Methods

Technology Comes of Age (1980s)

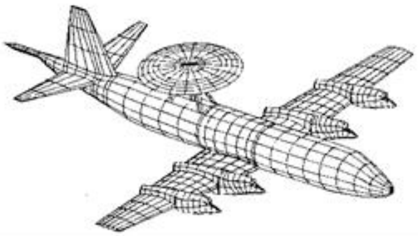
- **PANAIR (Boeing):** Magnus, Ehlers and Epton—*NASA CR 3251, April 1980*
    - Subsonic or supersonic flow; arbitrary bodies; higher order singularity distribution
- **MCAIR (McDonnell):** Bristow and Hawk—*NASA CR 3528, March 1982*
    - Subsonic flow; arbitrary bodies; constant source, quadratic doublet singularities
- **VSAERO (AMI):** Maskew—*NASA CR 166476, Dec 1982*
    - Subsonic flow; arbitrary bodies; piecewise constant doublet and source singularities
- **QUADPAN (Lockheed):** Youngren, Bouchard, Coopersmith, and Miranda—*AIAA 83-1827, July 1983*
    - QUADriletral PANel code: subsonic flow; arbitrary bodies; low-order constant source and doublet singularities

Wing-Body-Canard Analysis





P-3 AEW&C Development

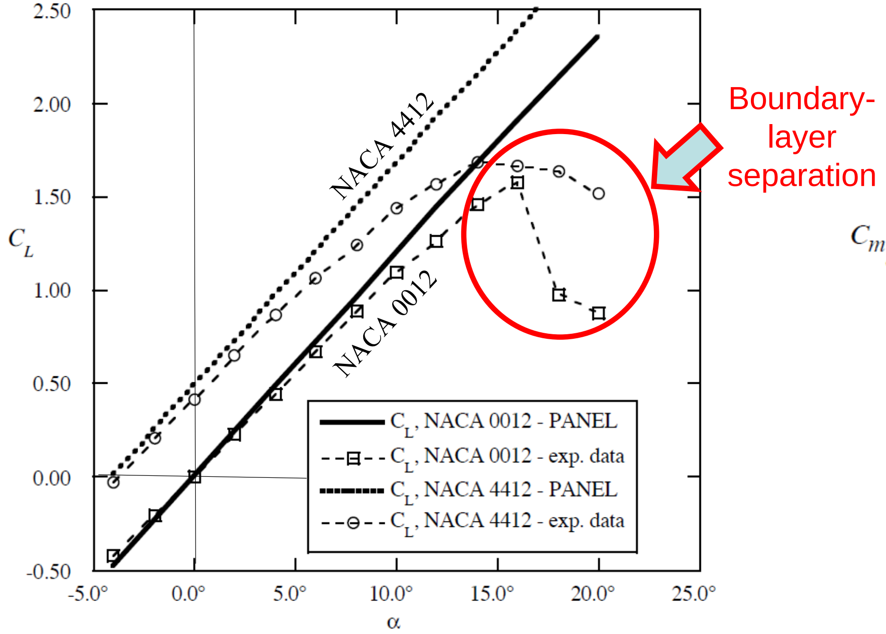


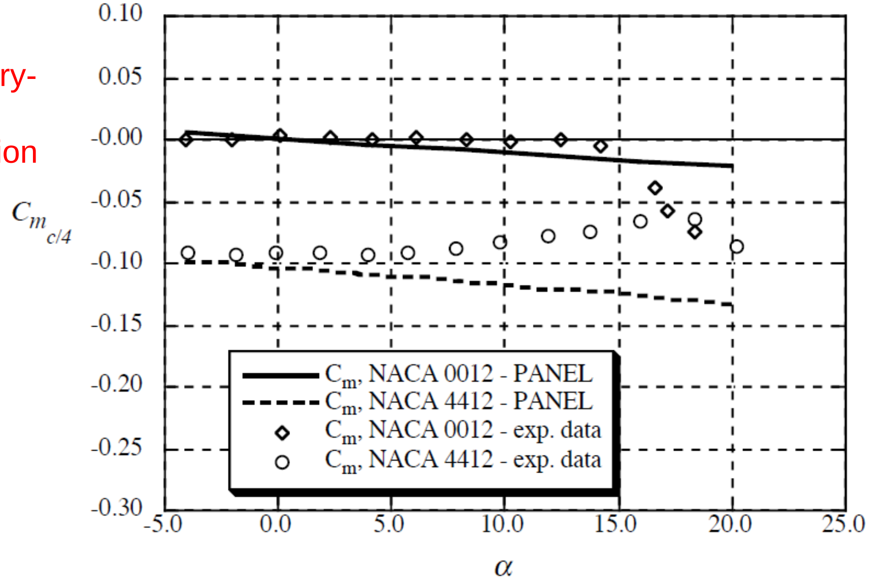
Applicable to Simulating Entirely Subsonic or Supersonic Attached Flows Only on Full Aircraft Configurations

Source: Refs. 5.1.13 – 5.1.29

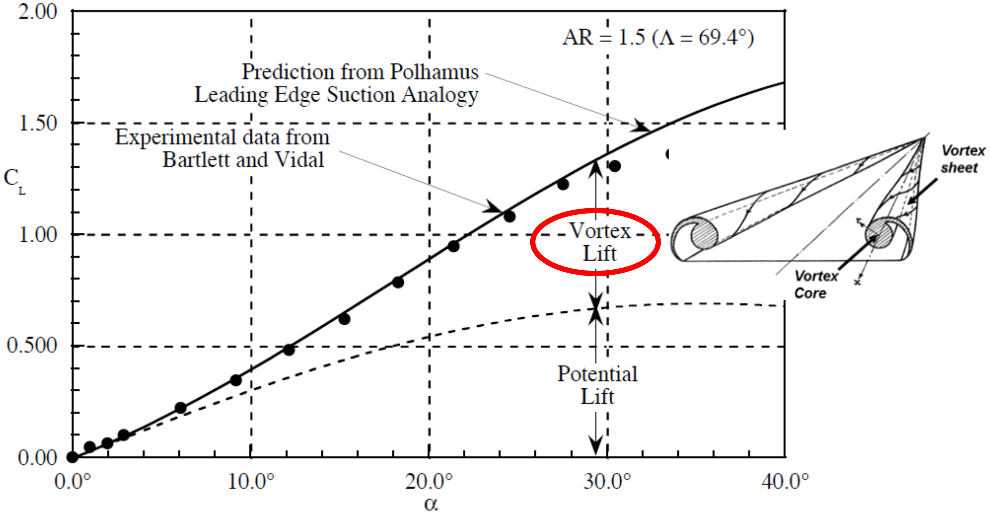
Limitations of LPM’s Applicability

Example 1: *Symmetric and Cambered Airfoils Flow Simulation*





Example 2: *Delta Wing Flow Simulation*



LPMs applicable to simulate

- ***attached*** **flows that are entirely subsonic or supersonic; *not* transonic**
- **flows *not dominated by* shocks, vortices, or boundary-layer separation**

Assessment Based on Comparing LPM Results With Experimental Data!

Source: Mason (Configuration Aerodynamics)

“Higher, Faster, Farther” Jet Transports

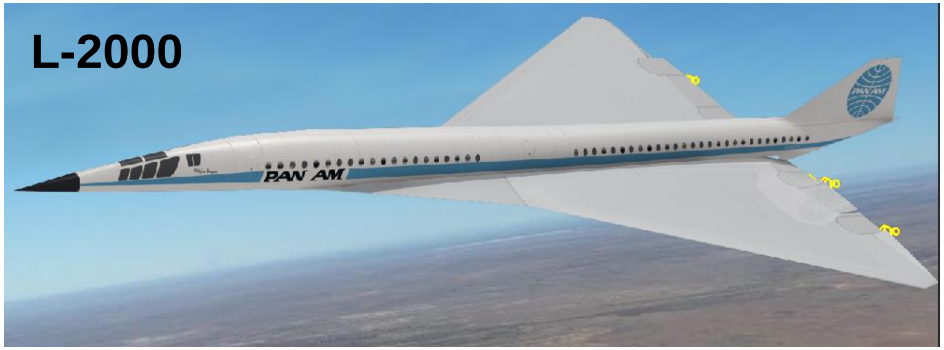
US SST (Supersonic Transport) Aircraft (*1960s*)

- **June 5, 1963: FAA launched the SST program to *improve upon the Anglo-French Concorde* with quite aggressive targets**
  - 250 passengers
  - $M_{cruise} = 2.7 – 3.0$
  - 4,000 miles Range

B 2707

- **January 15, 1964: Proposals submitted**
  - Boeing and Lockheed entries downselected for further development
  - Boeing developed swing-wing B 2707, and Lockheed L-2000

L-2000



- **January 1, 1967: Boeing won the competition**
- **May 20, 1971: Development work stopped; US Congress canceled funding**
  - Rising costs and lack of a clear market were likely factors

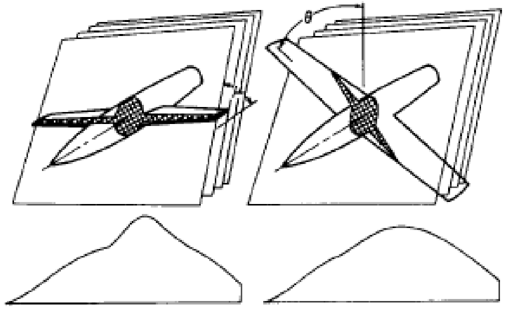
Design Needs of SST Stimulated Research in Many Areas

Image Source: Internet

"Computer-Aided Aerodynamics"

Leverage Computers to Meet SST Aerodynamic Design Needs (1960s)

- **Wave Drag Analysis–*Harris (1964)***
  - Analysis and correlation of aircraft wave drag—*NASA TM X-947*

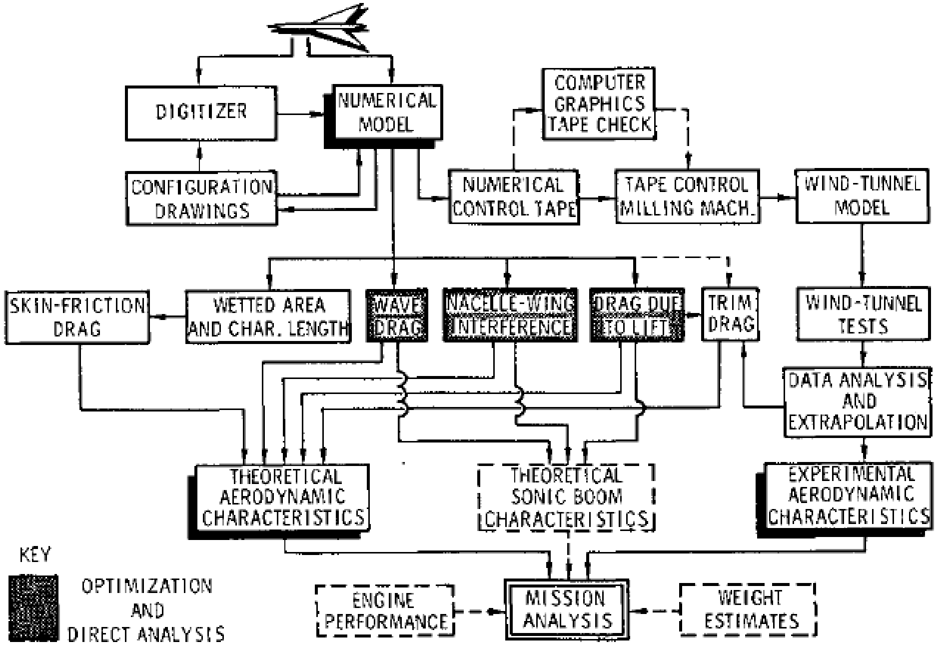


$$D(\theta) = -\frac{\rho V^2}{4\pi}\int_0^l\int_0^l A''(x_1)\,A''(x_2)\,\mathrm{LOG}\,|x_1 - x_2|\,dx_1\,dx_2$$

$$D = \frac{1}{2\pi}\int_0^{2\pi} D(\theta)\,d\theta$$

- **Supersonic Wing Camber Design – *Carlson and Middleton (1964)***
  - Numerical method for designing camber surfaces of supersonic wings with arbitrary planform corresponding to specified load distributions—*NASA TN D-2341*

- **Supersonic Aircraft Design Integration – *Baals et al (1968)***
  - Aerodynamic design integration of supersonic aircraft—*AIAA Paper 68-1018; also in Journal of Aircraft, 7(5), 1970*



"Computer-Aided Aerodynamics" Demonstrated Its Usefulness

Source: Refs. 5.1.30 through 5.1.32

“Long-Haul, High-Capacity” Jet Transports

Transonic Aircraft (*1960s*)

- **Jet transport designs in the 1960s pushed cruise speed into transonic regime to maximize Range Factor, M_{cruise} *(L/D)***
  - C-5A (1968): $M_{cruise} = 0.77$
  - B747 (1969): $M_{cruise} = 0.84 – 0.88$
  - L-1011 (1970): $M_{cruise} = 0.86$
- **Drag rises with speed due to added wave drag + shock-induced separation drag**
  - ***The higher the drag rise Mach number, the better!***
  - Sweep helps…but design tradeoffs limit it to about 35^o in practice

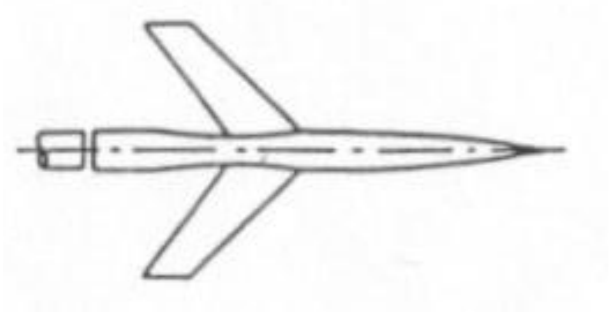
LPMs of Little Use for Accurate Transonic Flow Simulation

Image Source: Internet

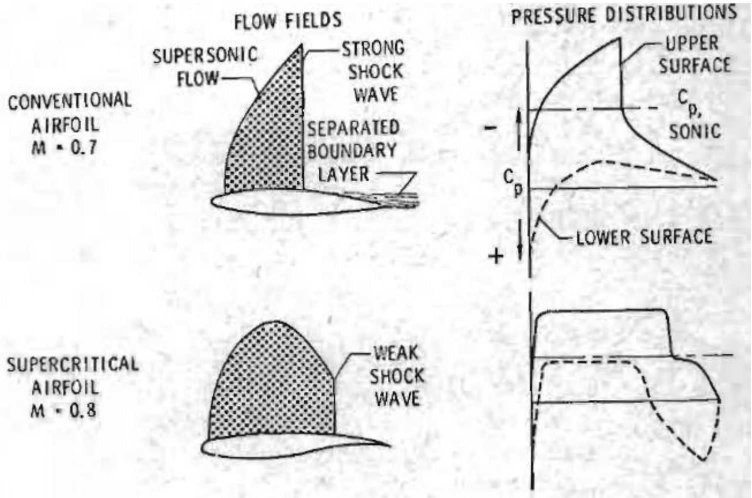
Transonic Aircraft Design

EFD: Primary Means of Flow Simulation

- **Whitcomb (1954 Collier Trophy)**
  - ○ "Area Rule"



- **Pearcy (1962)**
  - ○ "Peaky" airfoils: 0.02 to 0.03 increase in drag rise Mach number over NACA 6-series
- **Whitcomb (1967)**
  - ○ Supercritical "roof top" airfoils



ACA Capability Urgently Needed to Support Design Needs!

Source: Refs. 5.1.33 & 5.1.34

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Topic 5

5. Evolution of Applied Computational Aerodynamics *(1950-2000)*

5.1 Infancy through Adolescence (1950–1980)

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5.1.2 Rubbert, P.E., “Theoretical Characteristics of Arbitrary Wings by a Nonplanar Vortex Lattice Method,” Boeing Report D6-9244, The Boeing Company, 1964.

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