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A4

Air Vehicle Design

AOE 4065 – 4066

II. Air Vehicle Design Fundamentals

Course Module A4

Initial Sizing: *Takeoff Weight Estimation*

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AOE 4065-4066:
Capstone Air Vehicle Design (AVD) Course Modules (CMs)

Overview of AVD Courses

I. Foundational Elements

- **F1. Design: *An Engineering Discipline***
- **F2. Systems and Systems Thinking**
- **F3. Basics of Systems Engineering**
- **F4. Decision Making with Ethics and Integrity**

II. Air Vehicle Design Fundamentals

- **A1. Purpose & Process**

Conceptual Design

- **A2. Understand the Problem**
- **A3. Solve the Problem**
- **A4. Initial Sizing: *Takeoff Weight Estimation***
- **A5. Initial Sizing: *Wing Loading and Thrust Loading Estimation***
- **A6. Cost Considerations**
- **A7. Concept to Configuration: *Key Considerations***
- **A7A. Configuration Layout: *Drawings & Loft***

Conceptual & Preliminary Design

- **A8. Trade Studies**
- **A9. Use of Software Tools**
- **A10. Preliminary Design: *Baseline Design Refinement & Validation***

III. Project Management Topics

- **P1. Basics of Project Management and Project Planning**
- **P2. Project Organization**
- **P3. Roles & Responsibilities of Team Members**
- **P4. Project Execution: *Teamwork for Success***
- **P5. Project Risk Management**
- **P6. Delivering Effective Oral Presentations**
- **P7. Writing Effective Design Reports**

Disclaimer

Prof. Pradeep Raj, Aerospace and Ocean Engineering, Virginia Tech, collected and compiled the material contained herein from publicly available sources solely for educational purposes.

Although a good-faith attempt is made to cite all sources of the material, we regret any inadvertent omissions.

CRUCIALLY IMPORTANT

CMs only introduce key topics and highlight some important concepts and ideas…but without sufficient detail.

We must use lots of Reference Material* to add the necessary details!

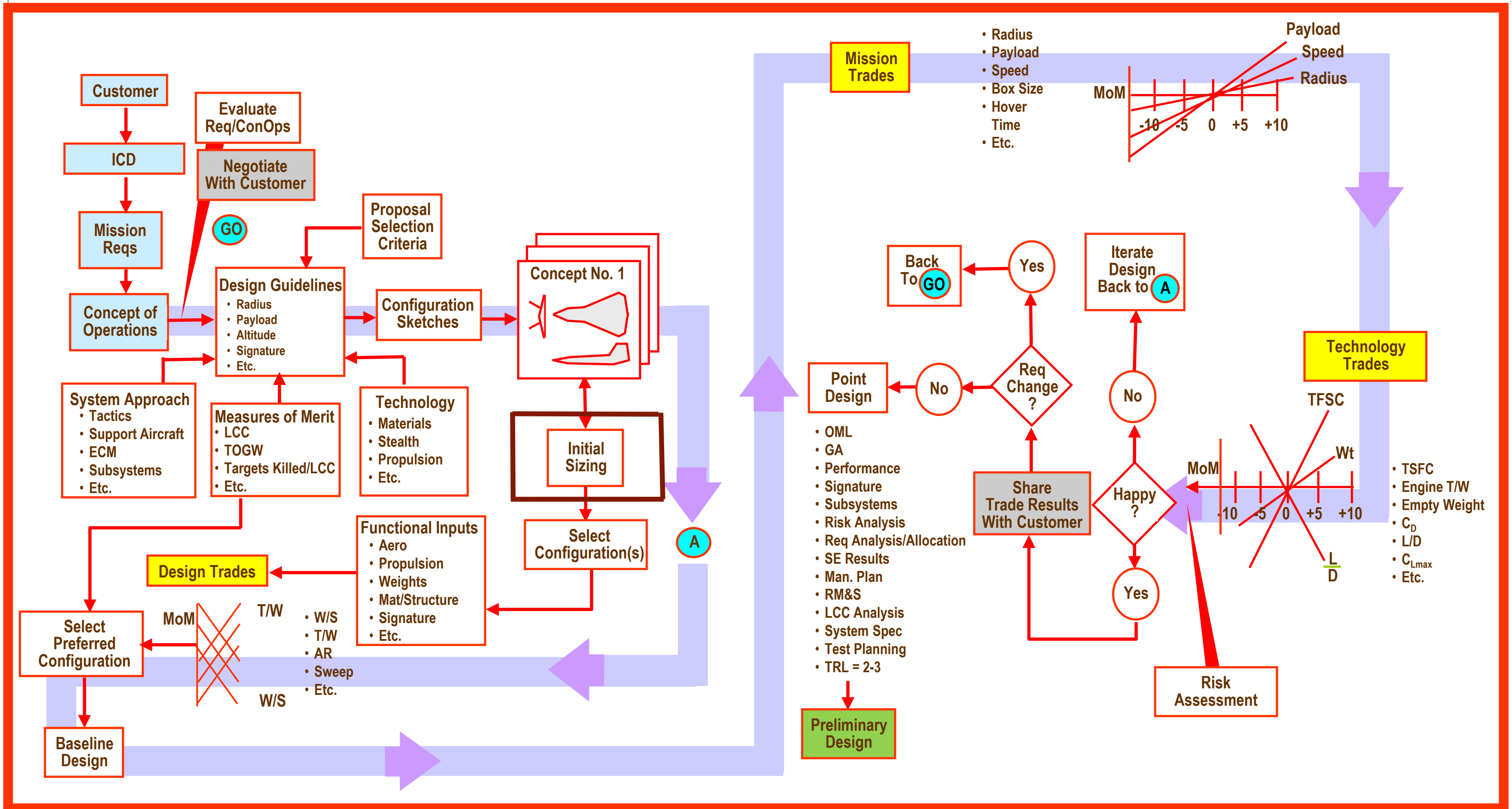
(*see Appendix in the Overview CM)

Outline

A4. Initial Sizing: *Take-off Weight Estimation*

*Internal Combustion Engine

Aircraft Conceptual Design (CD) Process



Initial Sizing: *The First Step for Selecting Feasible Concepts*

Adapted from Dr. Lee Nicolai's lecture slides

Initial Sizing

The 'Initial Sizing' element of the CD process is used to get the first estimate of aircraft weight, wing area, and thrust.

Initial Weight Sizing **(or initial take-off weight estimation) presented in this CM answers the question:**

How heavy is the airplane concept as sketched?

But, it doesn't tell us anything about the physical size of the airplane.

Two additional steps in the 'Initial Sizing' process, *Wing Loading* and *Thrust Loading* (covered in the next module) address the size aspects

i. Wing Loading provides an initial estimate of wing area required to produce the needed lift, and
ii. Thrust Loading provides an initial estimate of total thrust required to propel the aircraft

Importance of Airplane Weight

- Airplane Weight, W, is a very important design parameter.
- A quick glance at the following expression for power required in cruise, which is usually one of the most important flight segments of most missions, illustrates the significance of weight:

$$P_{req} = D \, . V = W \, . V \left\{ \frac{q \, C_{D_0}}{W/S} + \frac{W/S}{q(\pi A R e)} \right\}$$

Here D is total drag; V is flight speed; q is dynamic pressure ($0.5\rho V^2$); C_{D_0} is parasite drag coefficient; W/S is wing loading; AR is wing aspect ratio; and e is Oswald efficiency factor.

- The second term in the expression is directly proportional to the square of the airplane weight.

The Heavier the Airplane, the More Power it Needs to Fly!

Contributors to Takeoff Weight

- Take-off Weight, W_{TO} (also called Take-off Gross Weight, $TOGW$, W_{TOGW}) is W_{TO} is the sum of three types of weights: fixed weight, empty weight, and fuel (source of energy for propulsion) weight

$$W_{TO} = W_{fixed} + W_{empty} + W_{fuel}$$

- W_{fixed} includes non-expendables (crew, special equipment, sensors, etc.) plus payload (passengers, baggage) or expendables (cargo, bombs, missiles)

$$W_{fixed} = W_{nonexpendables} + W_{payload\ or\ expendables}$$

- W_{empty} includes airframe structure, engines, subsystems, avionics, etc., i.e., *everything needed to fly the airplane* except fuel and non-expendables and payload or expendables

- W_{fuel} is the weight of the fuel required to perform the mission (called mission fuel) <u>plus</u> reserve fuel per regulations (add approx. 5% of mission fuel at this stage) and trapped fuel (approx. 1% of mission fuel)

- The sum of W_{fixed} and W_{fuel} is also called 'useful' weight

$$W_{useful} = W_{fixed} + W_{fuel} = W_{TO} - W_{empty}$$

- W_{TO} is a good indicator of cost as well (cost/lb. is typically comparable *for a given class of aircraft*)

Outline

A4. Initial Sizing: *Take-off Weight Estimation*

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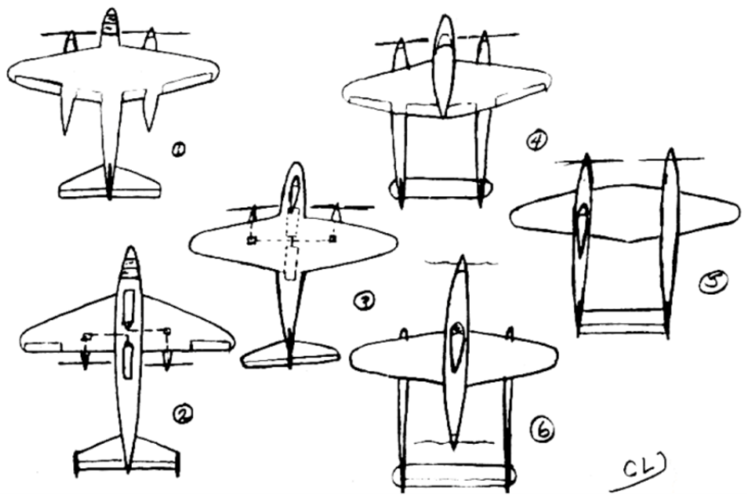
How to size a concept and determine IF it can perform the mission

- So, we have sketched our concepts.
- We think all of our concepts could perform the mission, i.e., *successfully fly to and from the destination with desired payload*.
- Now we just need to demonstrate that our concepts can, i.e., they are feasible.
- We can use Breguet Range Equation



$$R = \frac{V}{C}\frac{L}{D}\ln\frac{W_{i-1}}{W_i}$$

R = Range
C = Specific Fuel Consumption (*sfc*)
V = Speed
L/D = Lift-to-Drag Ratio
W_i = Ending Weight
W_{i-1} = Beginning Weight

- But we don't know V, L/D, C *(or sfc)*, and weight fractions (W_{i-1}/W_i)*?
- We could determine the values these parameters by analysis *if we knew the actual weight, engine, and airplane size and shape*. But we don't. Oops! Catch 22!

"Houston, we have a problem!"

**sometimes written as* (W_i/W_{i+1})

Estimating Take-Off Weight, W_{TO}

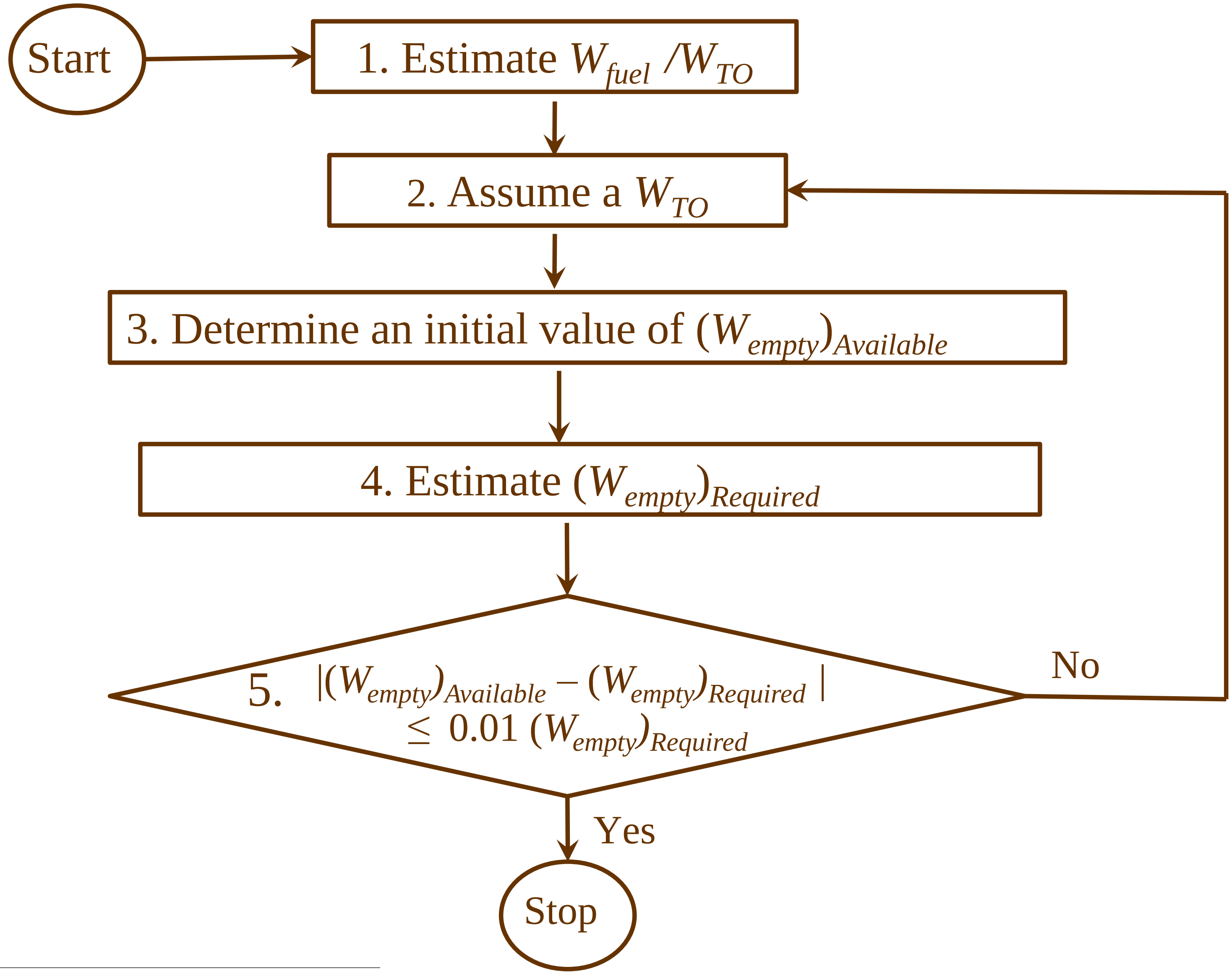
- It can be easily shown that

$$W_{TO} = \frac{W_{fixed}}{1 - \dfrac{W_{empty}}{W_{TO}} - \dfrac{W_{fuel}}{W_{TO}}}$$

- Therefore, given W_{fixed}, which is typically determined by mission requirements and ConOps, we can calculate W_{TO} if we knew the <u>empty weight fraction</u>, W_{empty}/W_{TO}, and the <u>fuel weight fraction</u>, W_{fuel}/W_{TO}
  We discuss ways of estimating them in this CM

- To get a first cut at W_{TO}, we *need to use an iterative procedure because W_{TO} is on both sides of the equation*

Flowchart of Iterative Procedure for Estimating W_{TO}



Five-step Iterative Procedure for Estimating W_{TO}

1. Estimate W_{fuel} / W_{TO} by piecemeal application of Breguet Range equation to all sections of the prescribed mission
2. Assume a value of W_{TO} (use historical data *for the class of vehicles to which your concept belongs*)
3. Determine an initial value of $(W_{empty})_{Available}$

$$(W_{empty})_{Available} = W_{TO} - W_{fuel} - W_{fixed}$$

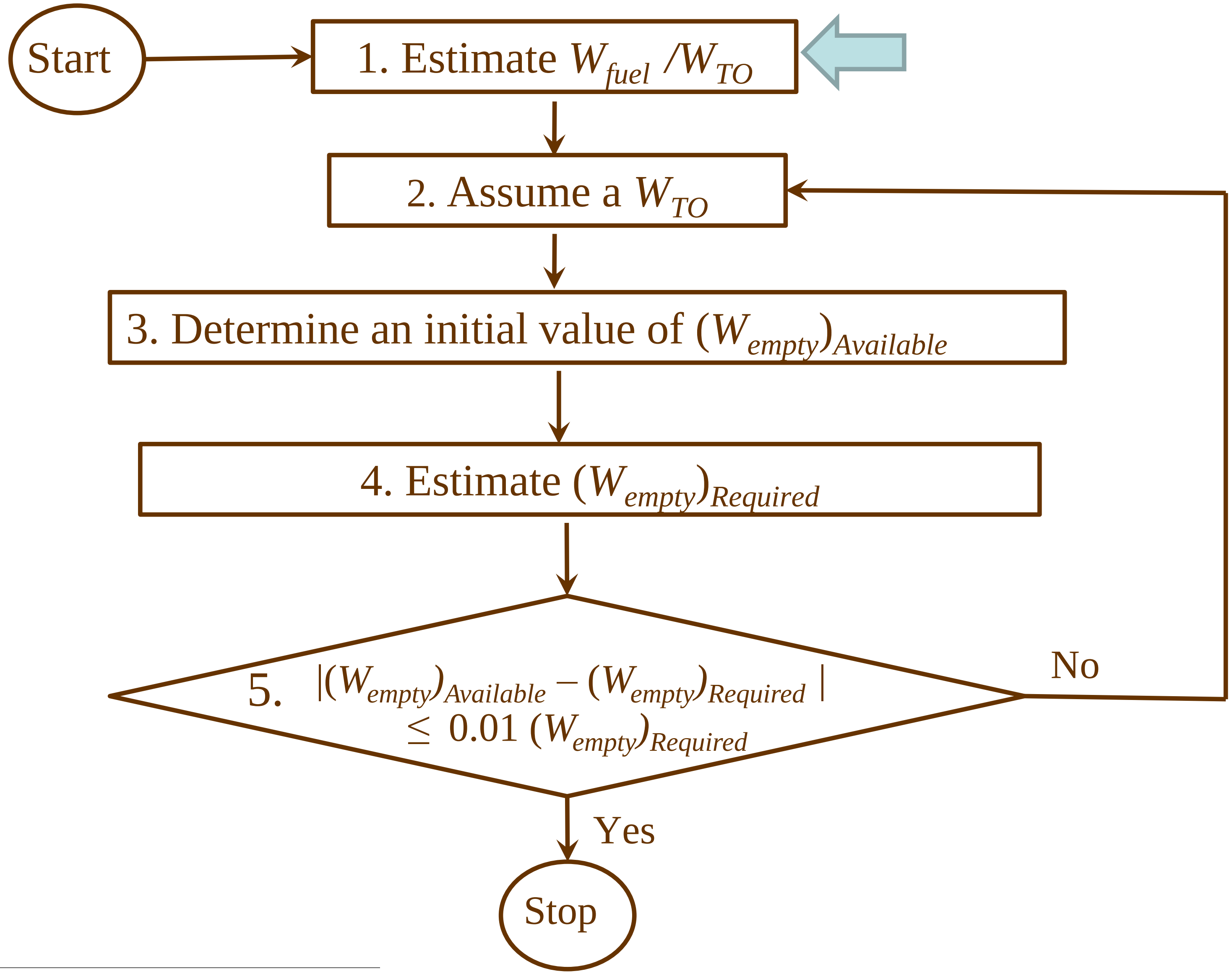
4. Estimate $(W_{empty})_{Required}$ (use historical data *for the class of vehicles to which your concept belongs*)
5. Iterate by varying the value of W_{TO} until the following convergence criterion is met

$$\left|(W_{empty})_{Available} - (W_{empty})_{Required}\right| \leq 0.01\,(W_{empty})_{Required}$$

Outline

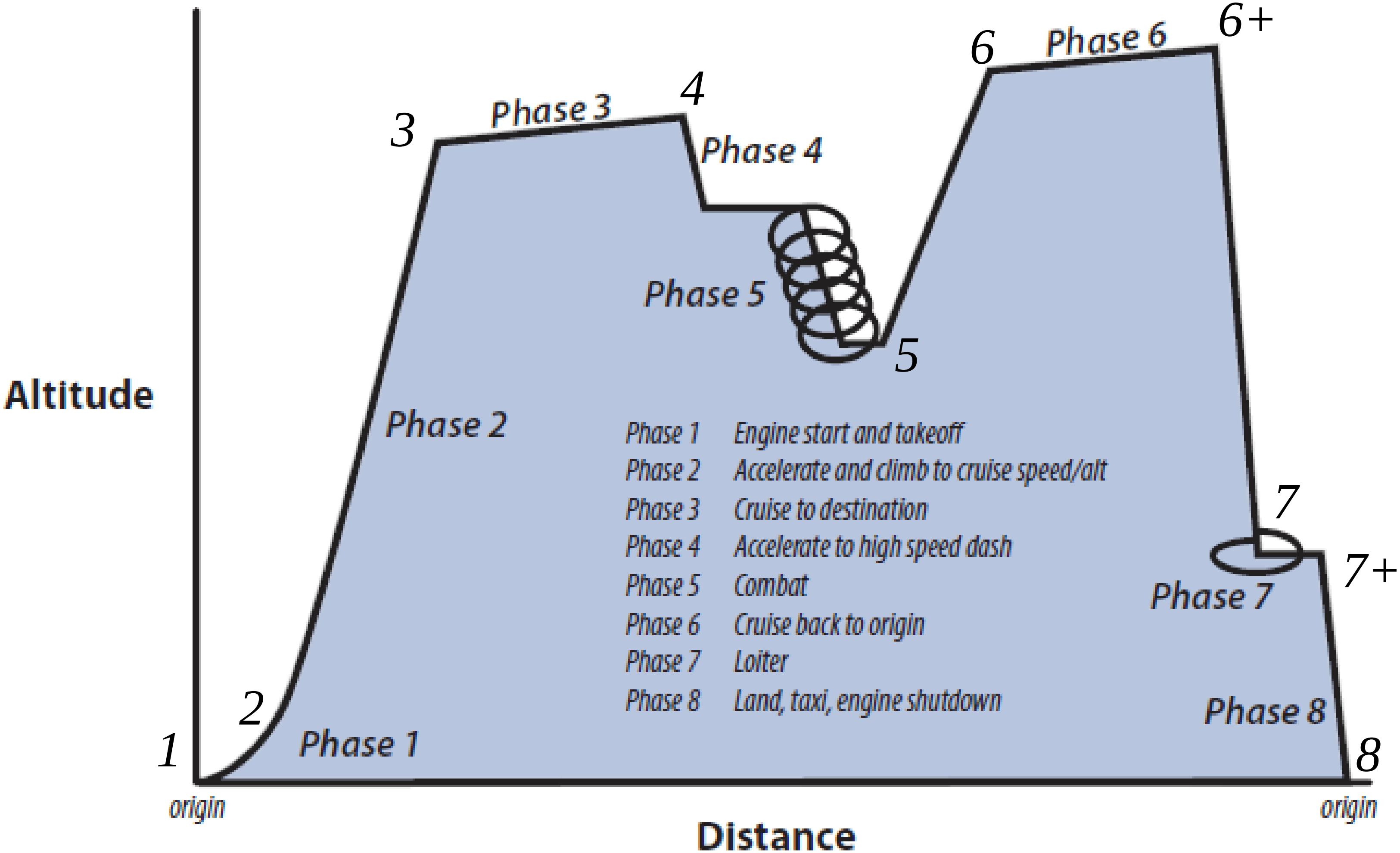
A4. Initial Sizing: *Take-off Weight Estimation*

Flowchart of Iterative Procedure for Estimating W_{TO}



Fuel Fraction, W_{fuel}/W_{TO} , Estimation

- A good Mission Profile is the starting point
- Let’s consider a notional eight-phase (or eight-segment) mission



Source: Fig. 5.1, Ref. AVD 1 (Nicolai and Carichner)

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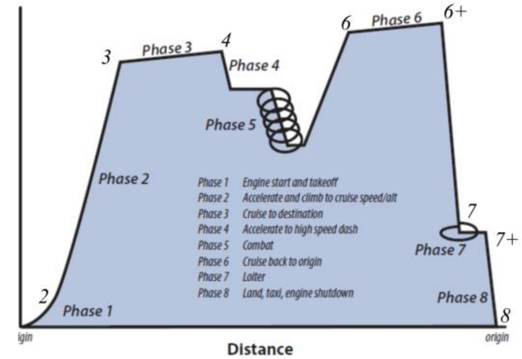
Fuel Fraction, W_{fuel}/W_{TO} , Estimation

- For a notional eight-phase (or eight-segment) mission:

$$W_{fuel} = \left(1 + \frac{W_{\substack{reserve\\fuel}}}{W_{TO}} + \frac{W_{\substack{trapped\\fuel}}}{W_{TO}}\right)\left(1 - \frac{W_8}{W_1}\right) W_{TO}$$

$$= \left(1 + \frac{W_{\substack{reserve\\fuel}}}{W_{TO}} + \frac{W_{\substack{trapped\\fuel}}}{W_{TO}}\right)\underbrace{\left(W_{TO} - W_{Landing}\right)}_{\text{ignores any other weight loss during mission}}$$



Mission Fuel Fraction

ASSUMPTION: weight change due to fuel consumption only

Same as $W_{Landing}$

$$\frac{W_{final}}{TOGW} = \frac{W_{final}}{W_{TO}} = \frac{W_8}{W_1} = \underbrace{\frac{W_2}{W_1}\frac{W_3}{W_2}\frac{W_4}{W_3}\cdots\frac{W_8}{W_7}}_{\substack{\text{weight fraction for each segment}\\\text{(must include a step change}\\\text{if you drop something)}}}$$

Assuming 5% reserve and 1% trapped fuel, the <u>Fuel Fraction</u> is:

$$\frac{W_{fuel}}{W_{TO}} = 1.06\left(1 - \prod_{i=1}^{7}\frac{W_{i+1}}{W_i}\right)$$

Fuel Fraction Estimation by Mission Phases (or Segments)

- For **all mission phases** ***<u>except</u>*** **cruise and loiter**, use the following estimates of W_{i+1}/W_i

Warmup and takeoff (Phase 1, W_2/W_1)	0.97
Climb & Accelerate* (Phase 2, W_3/W_2)	0.985
Combat (Phase 5, W_6/W_5)	*See Ref. AVD 1, pp. 130*
Descent	Part of Cruise
Landing (Phase 8, W_8/W_{7+})	0.995

- Fuel fraction for each segment can be obtained by subtracting W_{i+1}/W_i from 1. For example, the fuel fraction for Phase 1 is 0.03.
- We can also estimate the weight of the airplane at the end of any mission phase using the weight ratios. For example, once we have estimated the takeoff weight, the airplane weight at the end of phase 2 is given by

$$W_3 = (W_3/W_2)(W_2/W_1)W_1$$

- Note that supersonic aircraft (Concorde, SR-71, fighters) can burn much more fuel in climb & accelerate phase than others. Instead of using a fixed value of 0.985, use the procedure for Phase 4 in Section 5.4, Ref. AVD 1

Source: Table 3.2, Ref. AVD 2 (Raymer)

Fuel Fraction Estimation for Cruise and Loiter Mission Phases

- **Cruise** (Phase 3, W_4/W_3, and Phase 6, W_{6+}/W_6)

$$\frac{W_{i+1}}{W_i} = e^{-\frac{R \cdot sfc}{V(L/D)}}$$

R is cruise range

- **Loiter** (Phase 7, W_{7+}/W_{6+})

$$\frac{W_{i+1}}{W_i} = e^{-\frac{E \cdot sfc}{(L/D)}}$$

E is endurance, i.e., loiter time

- For **Cruise**, we can also write $V = a \cdot M$ where M is the flight Mach number and a is the speed of sound at the cruise altitude

- For **Cruise** and **Loiter** phases, we need estimates of
  (a) sfc
  (b) L/D
  (c) V (or M)

  to determine the fuel fraction, assuming R and E are specified.

How do we estimate these values?
Let's look into it next.

Source: Table 3.2, Ref. AVD 2 (Raymer)

(a) *sfc* or ***Specific Fuel Consumption*** estimation for Cruise and Loiter mission phases

Specific Fuel Consumption, *sfc*, Estimation for Cruise and Loiter Mission Phases

- ***Challenge:*** Determining *sfc (rate of fuel consumption per unit thrust)* would be pretty straightforward using engine data tables ***if we knew the engine*** (turbojet or turbofan or turboprop), flight altitude, flight speed, throttle setting, etc. *But we don't!*
- ***Approach:*** Since we don't know, let us ***make some reasonable assumptions that will have to be checked and updated later in the design process*** when we have a better definition of the vehicle. We consider 3 approaches.
- Note: smaller the *sfc*, the better!

Approach 1

- For jet engines, we can use nominal values from this table:

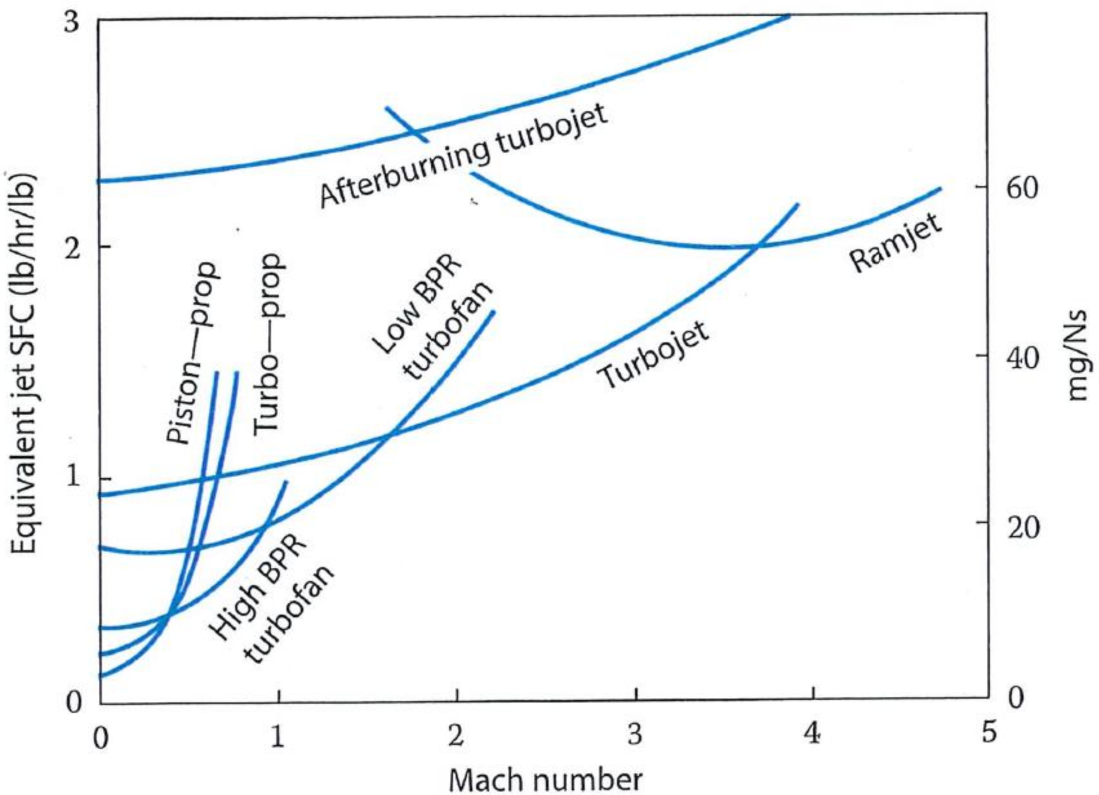
Typical Jet sfc (lb/hr/lb)	Cruise	Loiter
Pure turbojet	0.9	0.8
Low-bypass Turbofan	0.8	0.7
High-bypass Turbofan	0.5	0.4

Source: Table 3.3, Ref. AVD 2 (Raymer)

Specific Fuel Consumption, *sfc*, Estimation for Cruise and Loiter Mission Phases

Approach 2

- **Use Trend Charts** like the one shown on the right for *"typical cruise altitudes"*



- Beware that many times data and charts are for Uninstalled SLS (Sea Level Standard) conditions.

Need estimates of speed and altitude to estimate Mach number!

- Add 3-10% to account for complex installations or off-design conditions.

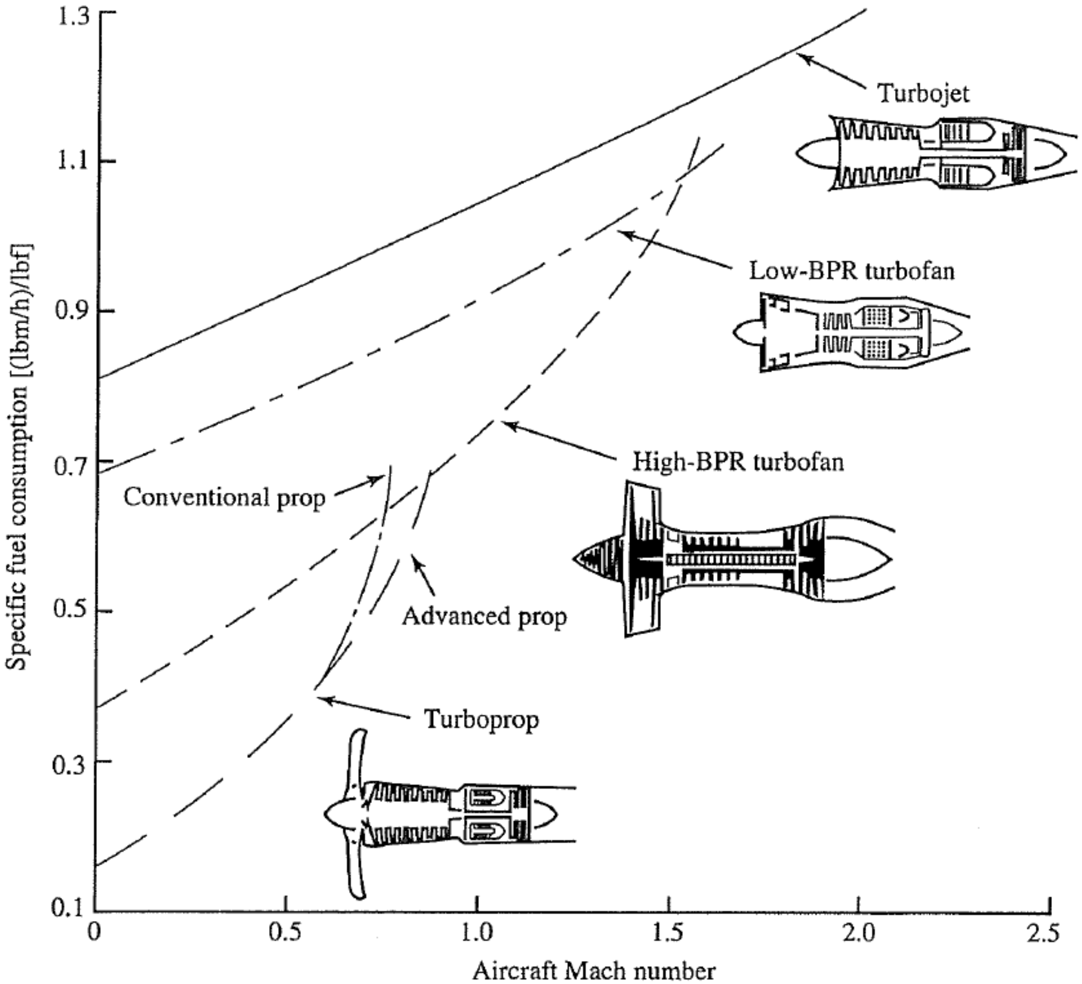
Source: Fig. 3.3, Ref. AVD 2 (Raymer)

Specific Fuel Consumption, *sfc*, Estimation for Cruise and Loiter Mission Phases

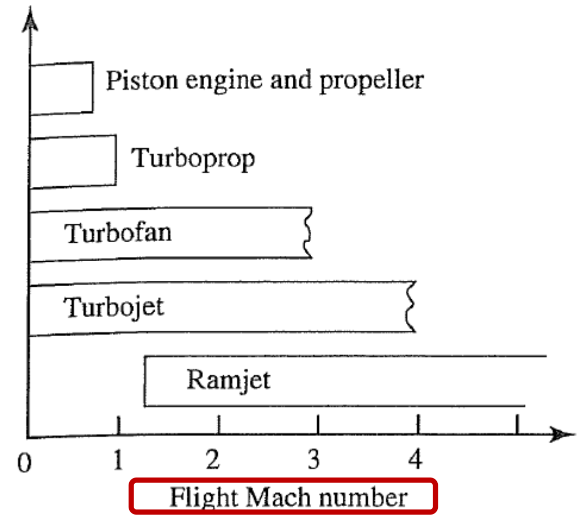
Approach 2 (contd.)

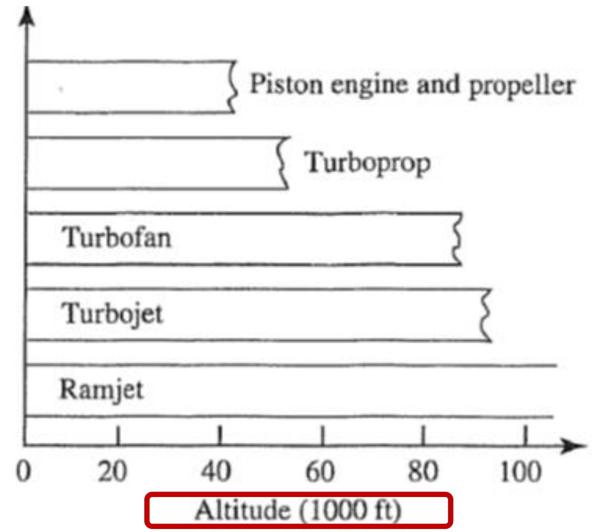
Use Trend Charts

Typical TSFC characteristics of engines



Engine Operational Limits





Source: Figs. 1.2 & 1.17b, Ref. PS 1 (Mattingly)

Specific Fuel Consumption, *sfc*, Estimation for Cruise and Loiter Mission Phases

Approach 3

- **At this stage of design, we can estimate *sfc* in *(lbm/hr)/lbf* using the following expressions even though the actual value of *sfc* (or *TSFC*) depends on engine cycle that we don't yet know.**
- **Turboprop** $sfc = (0.2 + 0.9M)\sqrt{\theta}$
- **High-bypass-ratio turbofan** $sfc = (0.4 + 0.45M)\sqrt{\theta}$
- **Low-bypass-ratio, mixed-flow turbofan**
  - Military and lower power settings $sfc = (1.0 + 0.35M)\sqrt{\theta}$
  - Maximum power setting $sfc = (1.8 + 0.30M)\sqrt{\theta}$
- **Turbojet**
  - Military and lower power settings $sfc = (1.0 + 0.35M)\sqrt{\theta}$
  - Maximum power setting $sfc = (1.7 + 0.26M)\sqrt{\theta}$
- **We need values of Mach number, M, and static absolute temperature ratio, $\theta = T/T_{SL}$, for the flight altitude to estimate *sfc* using these formulas.**

Source: Sec. 1.5.4, Ref. PS 1 (Mattingly)

(b) L/D or ***Lift-to-drag Ratio*** estimation for Cruise and Loiter mission phases

L/D Estimates for ***Most Efficient*** Cruise and Loiter Mission Phases

- **Most efficient or "best" cruise or loiter corresponds to *minimum fuel consumption* which maximizes cruise range or endurance (loiter time)**
- **For three typical flight conditions, estimates for *L/D* are:**

Maximum jet range, constant throttle	$\frac{L}{D}=\frac{\sqrt{C_{D_0}/2K}}{C_{D_0}+KC_{D_0}/2K}=\sqrt{\frac{2}{9C_{D_0}K}}=\sqrt{\frac{8}{9}}(L/D)_{max}=0.\mathbf{943}(L/D)_{max}$
Maximum jet range, constant altitude	$\frac{L}{D}=\frac{\sqrt{C_{D_0}/3K}}{C_{D_0}+KC_{D_0}/3K}=\sqrt{\frac{9}{48C_{D_0}K}}=\sqrt{\frac{3}{4}}(L/D)_{max}=0.\mathbf{866}(L/D)_{max}$
Maximum propeller endurance	$\frac{L}{D}=\frac{\sqrt{3C_{D_0}/K}}{C_{D_0}+K3C_{D_0}/K}=\sqrt{\frac{3}{16C_{D_0}K}}=\sqrt{\frac{3}{4}}(L/D)_{max}=0.\mathbf{866}(L/D)_{max}$

Maximum jet endurance requires *L/D* to be near $(L/D)_{max}$ **Note:** $K = 1/(\pi ARe)$

- **The derivations assume uncambered wing:** $C_D = C_{D0} + KC_L^2$ **as the drag polar**
- $(L/D)_{max}$ **corresponds to a** C_L **that minimizes the total drag,** *D.*
- **Estimated values of** C_L **(assuming uncambered wings) for maximum range or endurance for different flight conditions are shown on the next slide**

Source: Table 3.2, Ref. AVD 1 (Nicolai and Carichner)

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Prescribed C_L Values for Maximum Range or Endurance

Uncambered Wing			
Mission	**Condition**	**Maximize**	**Value of C_L[a]**
Range—jet	Constant altitude	$C_L^{1/2}/C_D$	$\sqrt{C_{D_0}/3K}$
Range—jet	Constant throttle	$C_L/C_D^{3/2}$	$\sqrt{C_{D_0}/2K}$
Range—propeller	Constant altitude	C_L/C_D	$\sqrt{C_{D_0}/K}$
Range—sailplane	Minimum glide angle	C_L/C_D	$\sqrt{C_{D_0}/K}$
Endurance—sailplane	Minimum rate of sink	$C_L^{3/2}/C_D$	$\sqrt{3C_{D_0}/K}$
Endurance—propeller	Minimum power required	$C_L^{3/2}/C_D$	$\sqrt{3C_{D_0}/K}$
Endurance—jet	Minimum thrust required	C_L/C_D	$\sqrt{C_{D_0}/K}$
	Use $C_D = C_{D_0} + KC_L^2$ to find L/D or C_L/C_D and $(L/D)_{max} = 1/(2\sqrt{C_{D_0}K})$		

- **For cambered wing, see Table 3.2, Ref. AVD 1**

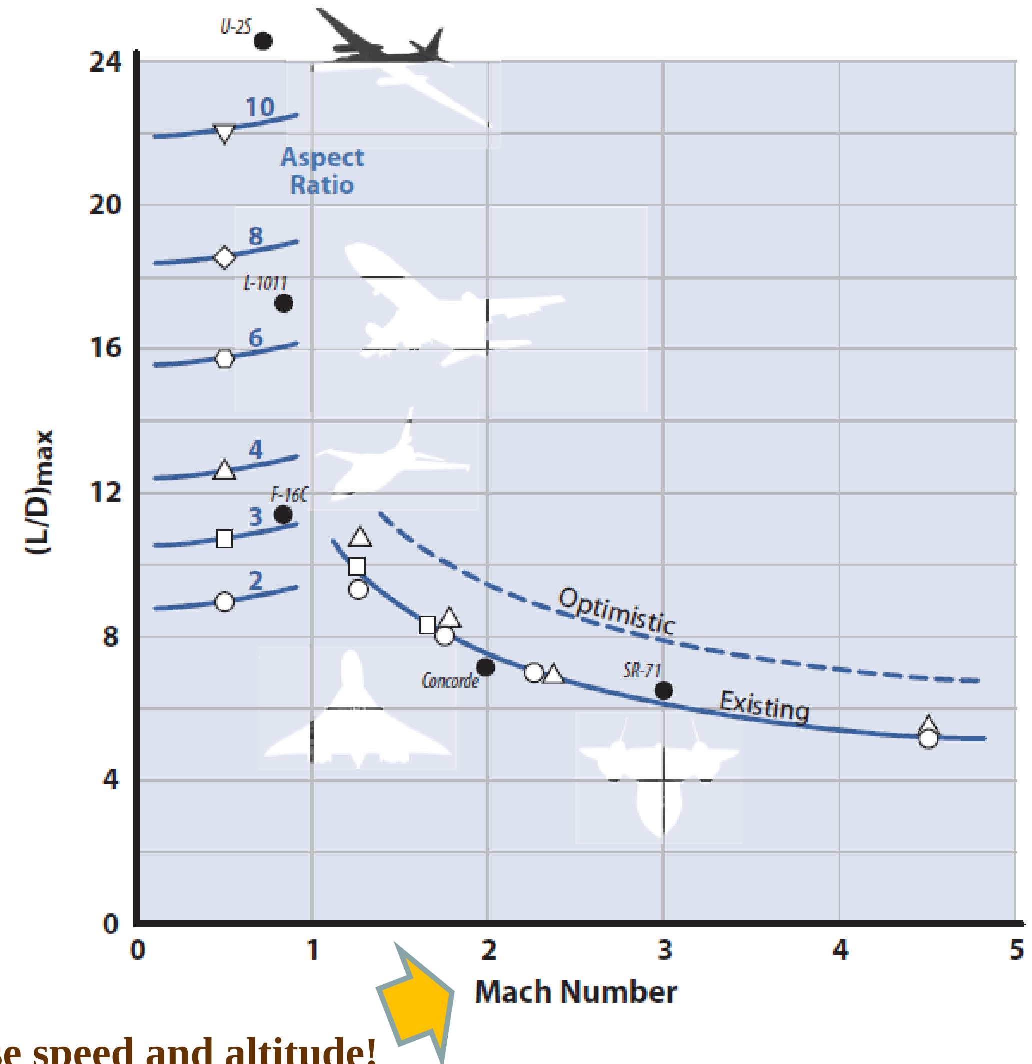
We still need to estimate *(L/D)$_{max}$* which is solely a function of configuration aerodynamics, *but the configuration is unknown at this stage!*

Let’s examine 3 approaches we could use.

Source: Table 3.2, Ref. AVD 1 (Nicolai and Carichner)

$(L/D)_{max}$ Estimation: Approach 1

Assume a wing AR and *estimate* $(L/D)_{max}$ using historical trend data



Need estimates of cruise speed and altitude!

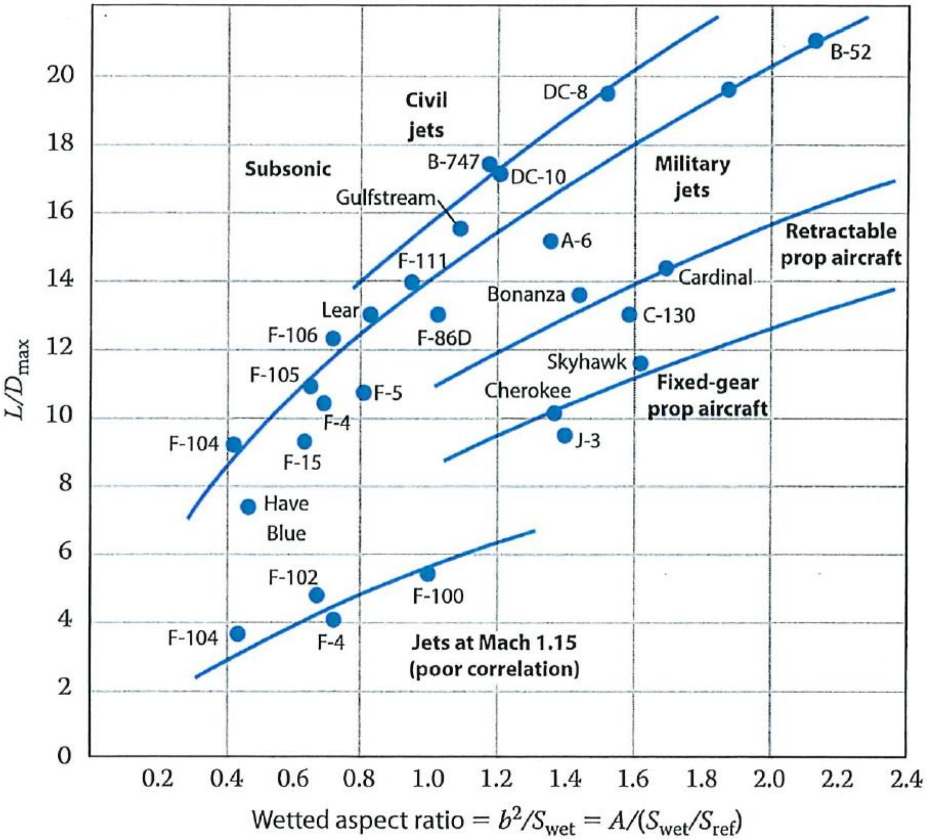
Source: Fig. 5.3, Ref. AVD 1 (Nicolai and Carichner)

$(L/D)_{max}$ Estimation: Approach 2

- First estimate *Wetted Aspect Ratio*, defined as $AR / (S_{wet}/S_{ref})$
- S_{wet} is the *total* surface area of the aircraft exposed ("wetted") to the air, and S_{ref} is the wing reference area—*neither is known at this stage!*
- Two options for (S_{wet}/S_{ref}) estimation

Option 1: use rough estimates

Configuration	S_{wet}/S_{ref}
Flying Wings	2.1 – 2.5
Wing/Body/Tail (Clean)	3.2 – 6.0
Wing/Body/Tail (F-15)	5.0
Wing/Body/Tail (Airliner)	6.0 – 7.0

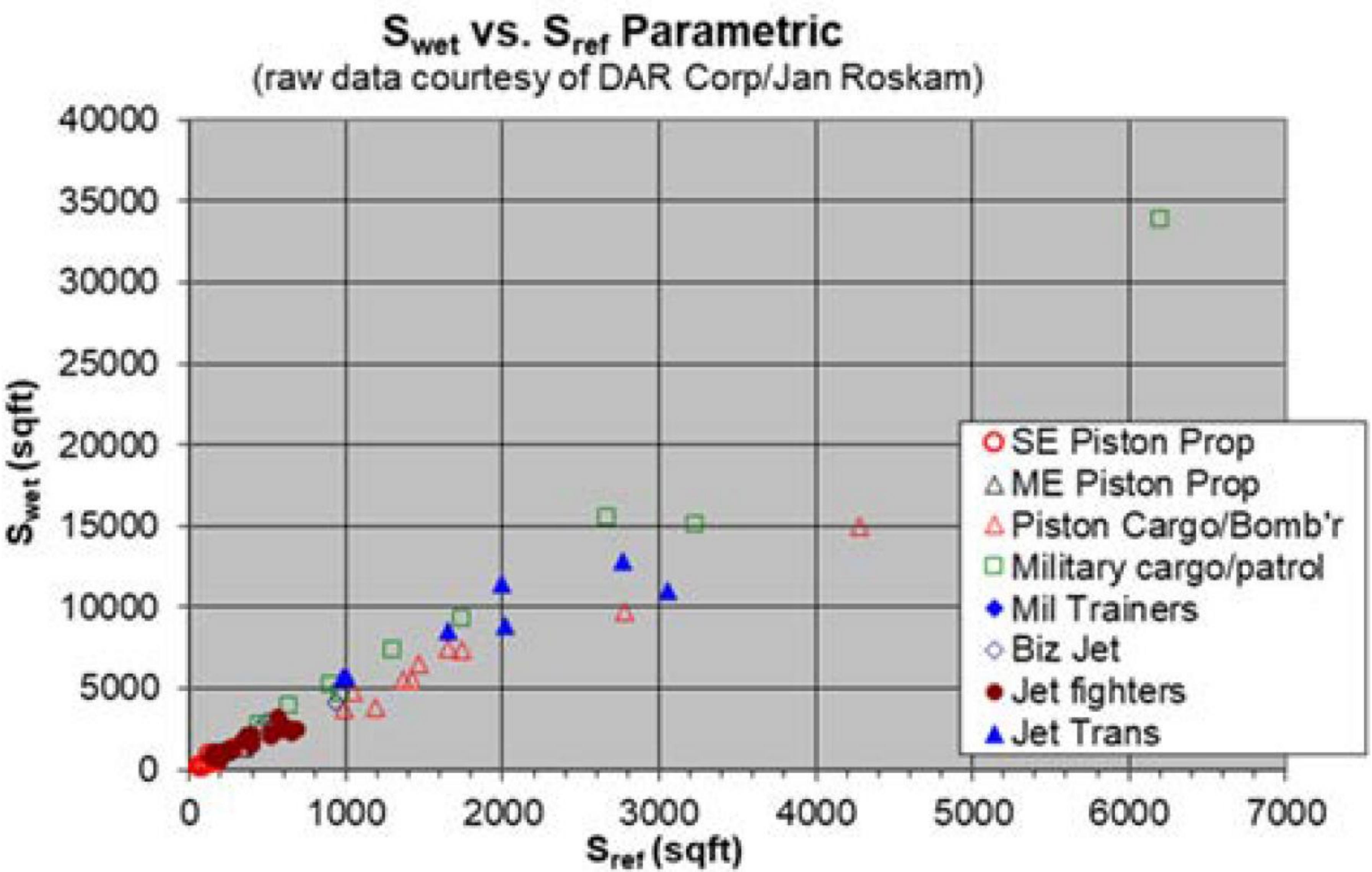


Source: Fig. 3.5, Ref. AVD 2 (Raymer)

- Use historical trend data *(shown in the figure above)* to estimate $(L/D)_{max}$ for your class of vehicle cpnfigruation

Option 2: use parametric correlations for different classes of vehicles *(see the next slide)*

$(L/D)_{max}$ Estimation: Approach 2 (contd.)



$(L/D)_{max}$ Estimation: Approach 3

Estimate $(L/D)_{max}$ using K and C_{D_0}

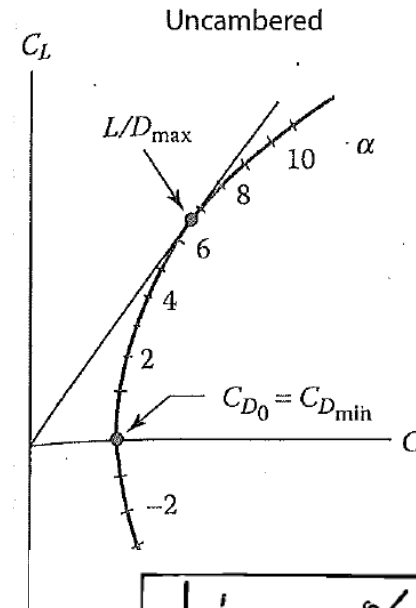
$$(L/D)_{max} = 1/\sqrt{4C_{D_0}K} = 0.5\sqrt{(\pi ARe)/C_{D_0}}$$

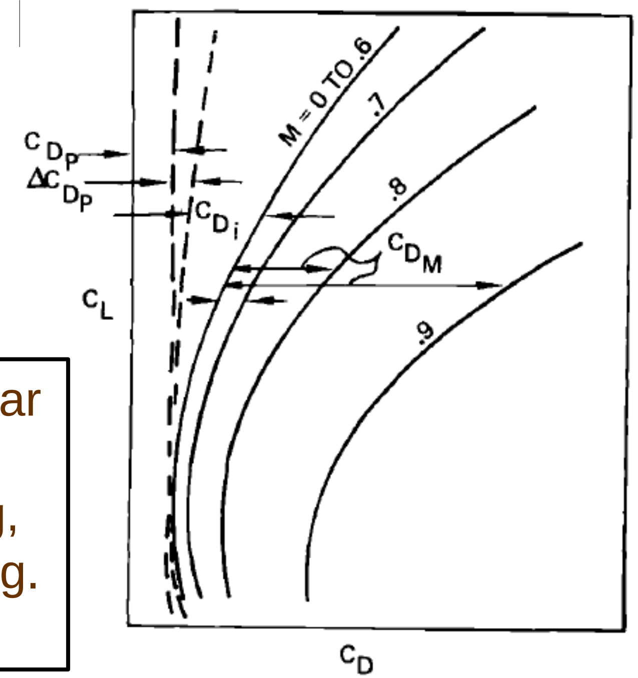
where

- C_{D_0} = zero-lift drag coefficient
- K = drag-due-to-lift factor, $K = 1/(\pi ARe)$
- AR = aspect ratio, b^2/S_{ref}
- b = span
- S_{ref} = wing reference area
- e = wing efficiency factor

$(L/D)_{max}$ is entirely a function of airplane aerodynamics!

For transonic flights, regions of supersonic flow appear when flight Mach number exceeds critical Mach number, i.e., $M > M_{crit}$. They produce additional drag, ΔC_{D_0}, due to compressibility which is called wave drag. It is highly desirable to keep it below 20 counts.





Source: Chapter 2, Ref. AVD 1 (Nicolai and Carichner)

$(L/D)_{max}$ Estimation: Approach 3 *(Step 1 of 2)*

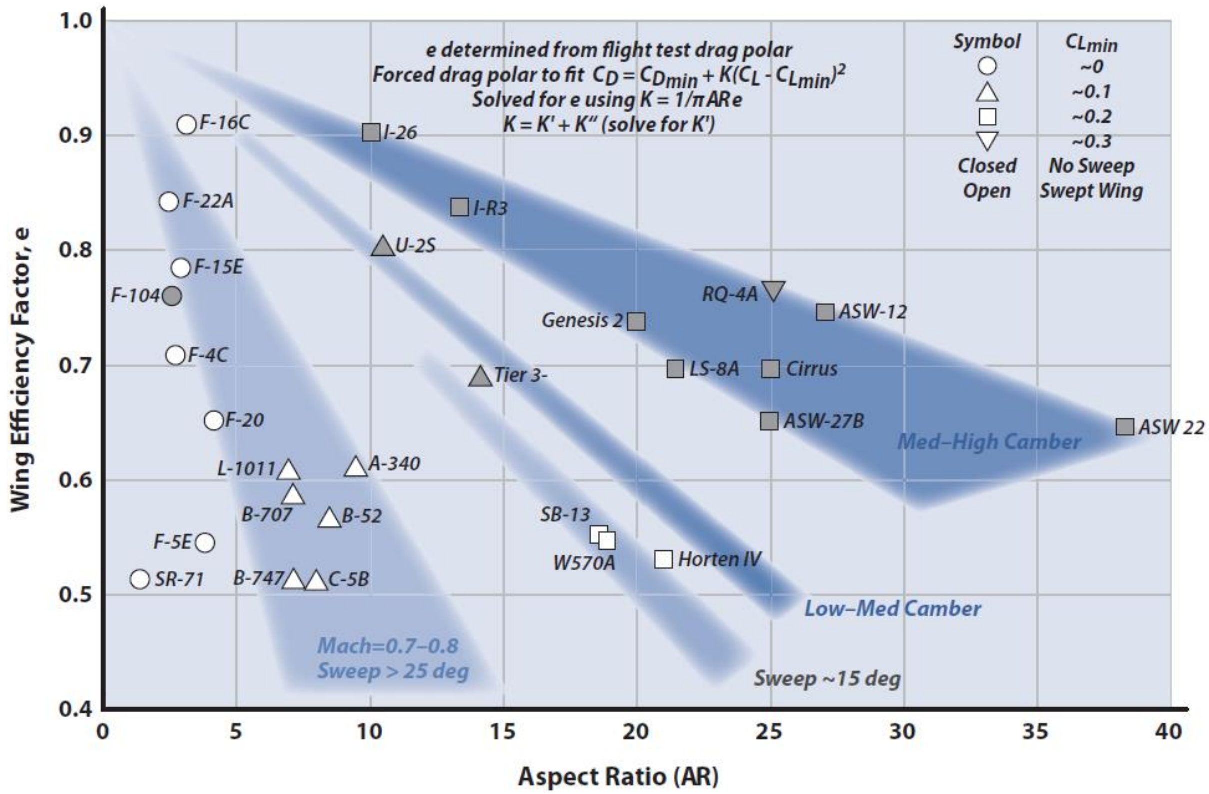
Step 1: Compute K using assumed AR and estimated e

$$K = 1/(\pi \cdot AR \cdot e)$$

Estimate wing efficiency factor, e, using a correlation chart of historical data shown on the right.

OR

Estimate wing efficiency factor, e, from a curve fit



Straight Wings: $e = 1.78(1 - 0.045AR^{0.68}) - 0.64$ for "normal" ARs $(4 < AR < 12)$

Swept Wings: $e = 4.61(1 - 0.033AR^{0.53})(\cos\Lambda_{LE})^{0.1} - 3.3$ for $\Lambda_{LE} > 30^{o}$ and "normal" ARs $(4 < AR < 12)$

Source: Fig. G.9, Ref. AVD 1; Equation, Ref. GED 2

$(L/D)_{max}$ Estimation: Approach 3 *(Step 2 of 2)*

Step 2. Estimate C_{D_0} using historical data (table below) and use it with K to determine $(L/D)_{max} = 1/\sqrt{4C_{D_0}K}$

Aircraft Type	Subsonic C_{D_0}
High subsonic jet transport	0.014-0.02
Supersonic fighter aircraft	0.014-0.022
Blended wing-body (tailless) jet aircraft	0.008-0.014
Large turboprop aircraft	0.018-0.024
Low-altitude subsonic cruise missile (high W/S)	0.03-0.04
Small single-engine propeller aircraft	
Retractable gear	0.022-0.030
Fixed Gear	0.026-0.04
Agricultural aircraft	
With spray system	0.07-0.08
without spray system	0.06
High-performance sailplane	0.006-0.01

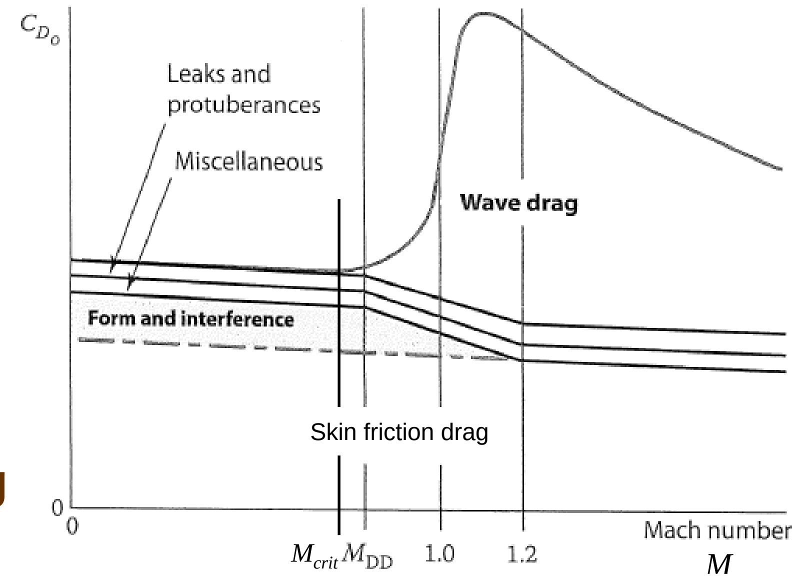
Source: Table 5.2, Ref. AVD 1 (Nicolai & Carichner)

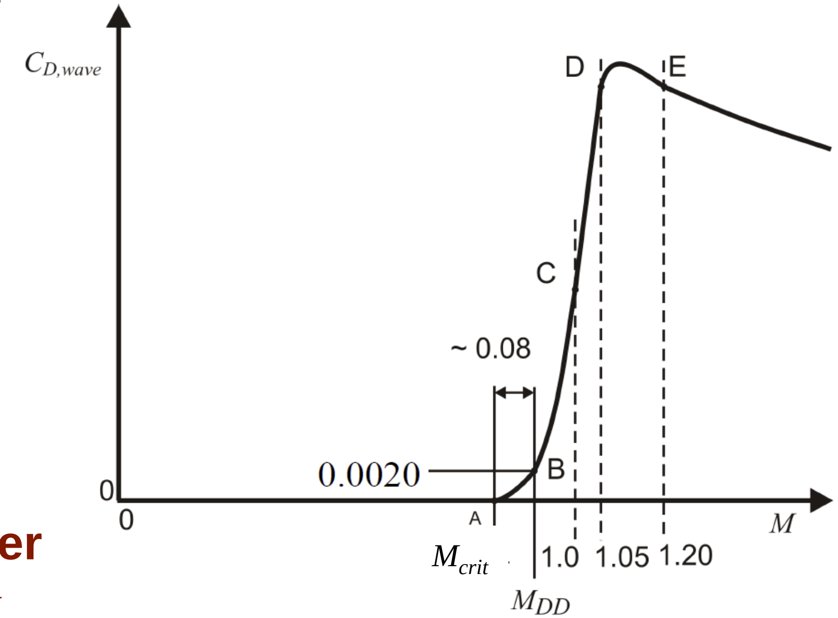
C_{D0} Variation with Mach Number

- **Airplane parasite drag coefficient, C_{D0}, for subsonic flight may be considered independent of Mach number, M, *until critical Mach number, M_{crit}, is reached*.**
- **Critical Mach number, M_{crit}, is the flight Mach number where the local flow becomes sonic at some point on the wing**
- **For flight Mach numbers $M > M_{crit}$, wave drag due to compressibility leads to "*drag rise*" *(see top right figure)***
- **The *additional* wave drag *(see bottom right figure)* may be estimated using an empirical relation:**

$$C_{D_{wave}} = 20\,(M - M_{crit})^4$$

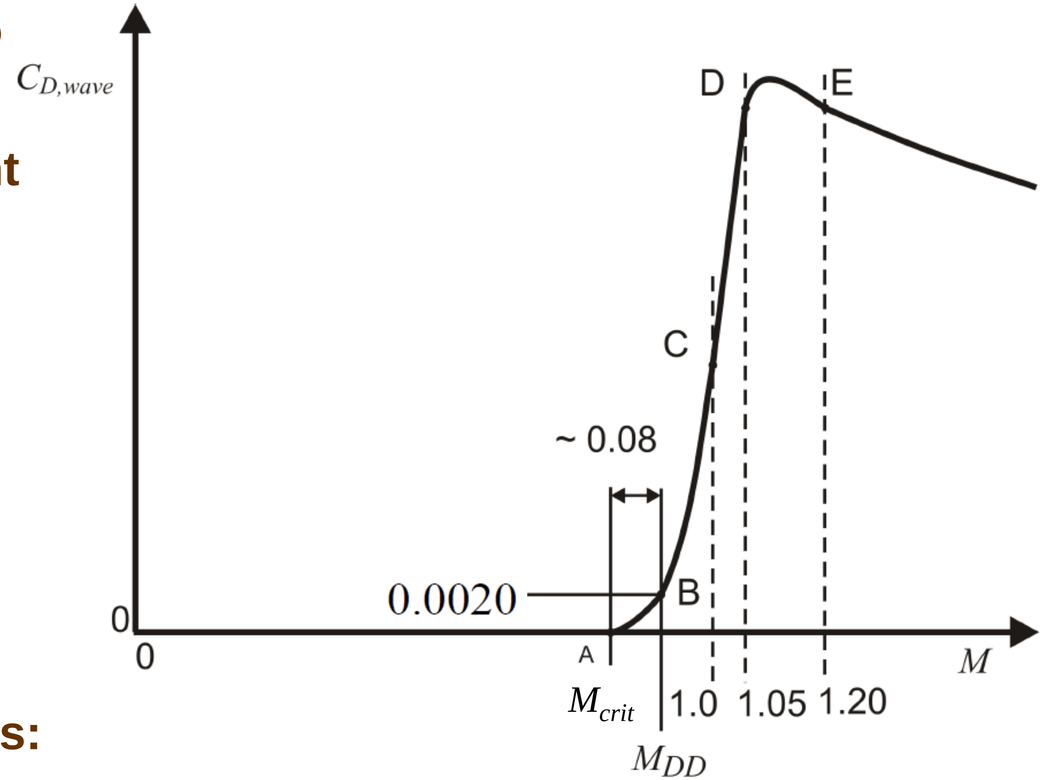
- **It is advisable to keep flight Mach number below drag divergence Mach number, M_{DD}**





Source: Chapter 7, https://www.fzt.haw-hamburg.de/pers/Scholz/HOOU/AircraftDesign_Contents.pdf

Drag Divergence Mach Number, M_{DD}



- **To keep the wave drag (due to compressibility) from exceeding 20 counts, the flight Mach number should not exceed drag divergence Mach number,** M_{DD}

- M_{DD} **corresponds to a flight Mach number where**

$$\partial C_D/\partial M = 0.1$$

- M_{DD} **is related to** M_{crit} **as follows:**

$$M_{DD} = M_{crit} + (0.1/80)^{1/3}$$

This is based on an empirical expression for $C_{D_{wave}}$ **and** M *(see previous slide)*

- **Another approximate expression relating** M_{crit} **and** M_{DD} **is:**

$$M_{DD} = M_{crit}\,[1.02 + 0.08(1 - \cos \Lambda_{c/4})]$$

Source: Chapter 7, https://www.fzt.haw-hamburg.de/pers/Scholz/HOOU/AircraftDesign_Contents.pdf

M_{DD} Depends on Airfoil Technology

- **Efficient transonic cruise depends critically on airfoil performance.**
- **For 2D airfoils, drag divergence Mach number is given by the Korn equation:**

$$M_{DD_{2D}} = \kappa_A - 0.1C_{L_{2D}} - (\mathrm{t/c})_{2D}$$

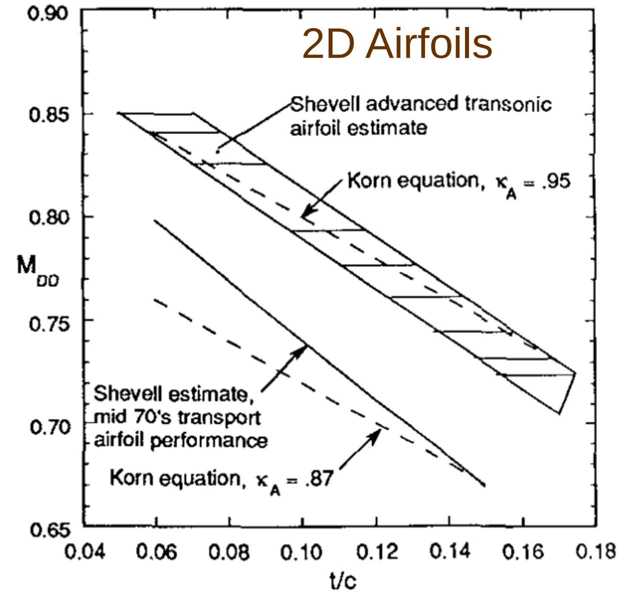
κ_A = 0.95 for NASA supercritical airfoils

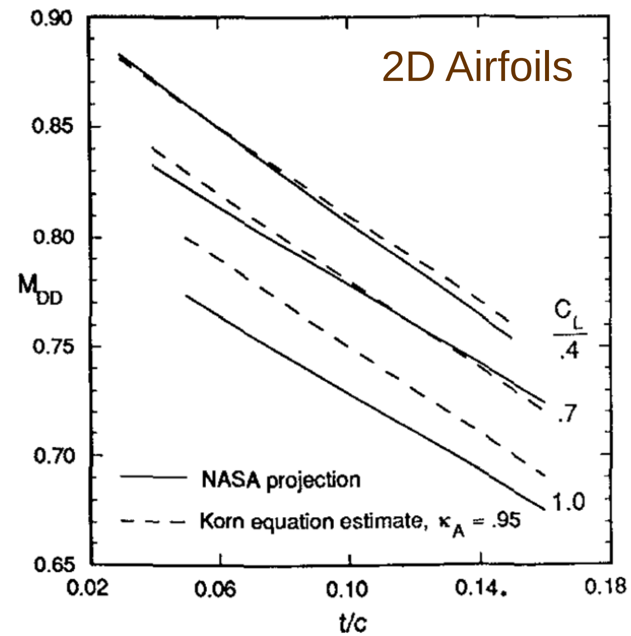
κ_A = 0.87 for conventional 6-series airfoils

- **For 3D wings, the following relation for drag divergence Mach number can be obtained using simple sweep theory:**

$$M_{DD} = \frac{\kappa_A}{\cos\Lambda} - \frac{t/c}{\cos^2\Lambda} - \frac{C_L}{10\cos^3\Lambda}$$

This relation permits investigation of the relative importance of wing sweep, Λ, and thickness ratio, t/c, for a chosen airfoil technology for a desired target flight Mach number and lift coefficient





(c) Speed (V) or Mach number (M) estimation for Cruise and Loiter mission phases

Speed (V) or Mach number (M) Estimation for Cruise and Loiter Mission Phase

- **Even if specified in RFP, it's instructive to estimate**
  - **Best Cruise Speed (or Best Cruise Mach, BCM) and Best Cruise Altitude (BCA) that give *maximum range for the least amount of fuel consumed;* and**
  - **Best Loiter Speed and Altitude to yield *maximum endurance for the least amount of fuel consumed***
- **Flying at best cruise speed and altitude maximizes *Specific Range* (sr) defined as *distance flown per pound of fuel*, which can be expressed as**

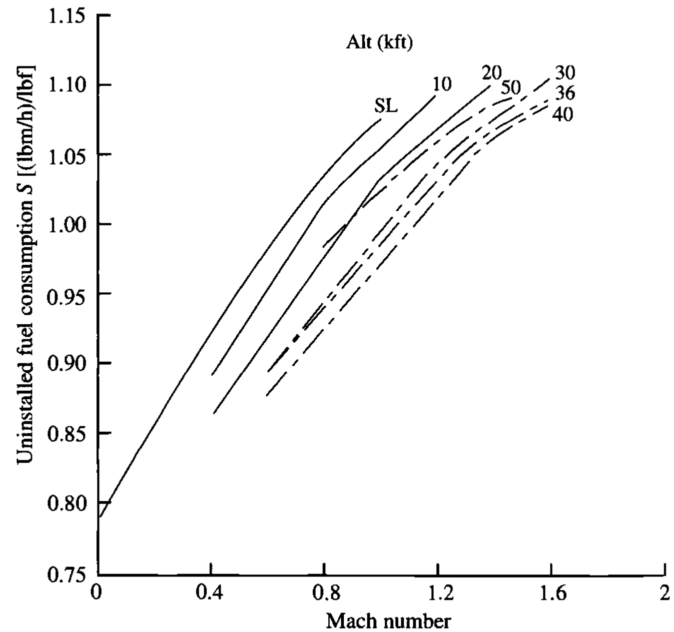
$$sr = \frac{V}{sfc}\frac{L}{D}\frac{1}{W} \qquad Range\ Factor,\ RF = \frac{V}{sfc}\frac{L}{D}$$

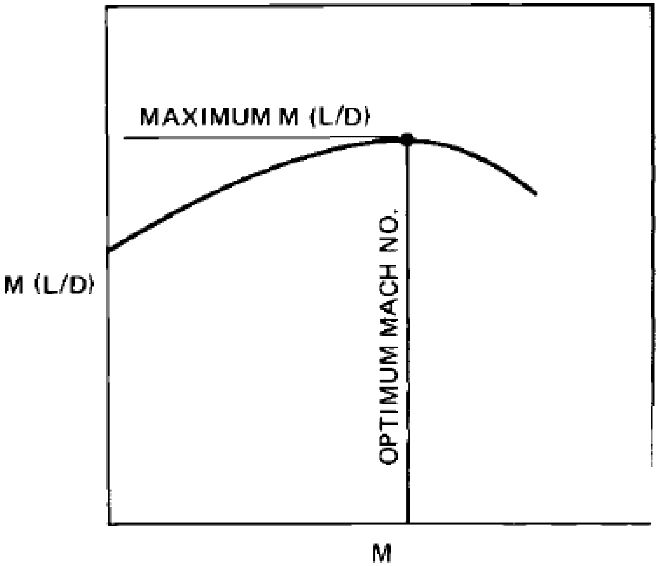
  - **For cruise flight conditions that give a nearly constant Range Factor, RF, integrating sr from the initial to the final condition yields the *Breguet Range equation.***
  - **For RF to be a constant, L/D and V/sfc should be constant *throughout the cruise flight*, which can be achieved using a *cruise climb* flight profile**
- **Best loiter speed and altitude maximize *Endurance Factor,* EF, defined as minimum fuel consumption for specified loiter time, which can be expressed as**

$$Endurance\ Factor,\ EF = \frac{1}{sfc}\frac{L}{D}$$

Speed *(V)* or Mach number *(M)* Estimation for Cruise Mission Phase

- **At a given Mach number (or speed), *sfc* of typical engines *decreases with altitude and reaches a low value between* 30,000 *and* 40,000 *ft.,* before increasing again as shown in the figure on the right.**
- **For altitudes > 36 *kft*, *sfc* may be considered nearly constant for a given Mach number, *M*, since θ is constant; *it still varies with M***



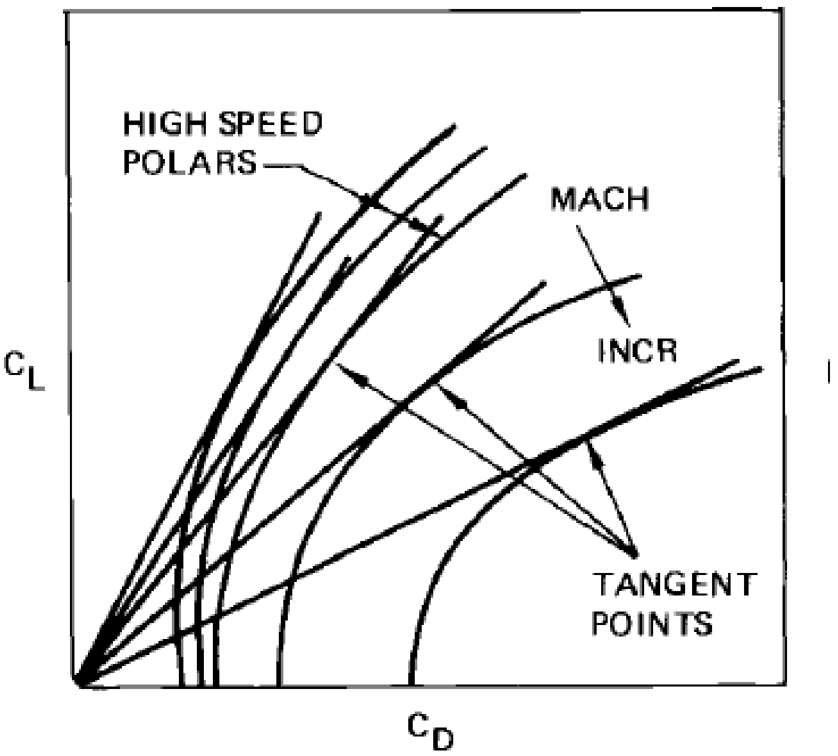


- **Then, Range Factor, *RF*, is maximized by maximizing *V*L/D* or *M*L/D***
- **Shouldn't *M*L/D* continue to increase with *M* indefinitely for a fixed *L/D*?**
- ***Then, why is there a peak followed by a drop in the figure on the left?***

Source: Fig. 1.14d, Ref. PS 1 (Mattingly); Fig. 3.22, Ref. PS 2 (Oates)

Speed (V) or Mach number (M) Estimation for Cruise Mission Phase

- **In reality, Range Factor, RF, reaches a maximum around M_{DD} and then decreases because of rapid increase in drag due to compressibility for $M > M_{DD}$ *(see slide 34)***
- **Typical drag polars *(shown in the figure on the right)* exhibit the effect of compressibility on $(L/D)_{max}$ and hence on L/D**
- **A tangent point on the drag polar for each Mach number, *M*, indicates the location of $(L/D)_{max}$ where lift-to-drag ratio, L/D, is maximum**



- ***Since the configuration hasn't been defined yet, we don't have actual drag polars at this stage of design! But, we have derived expressions for $(L/D)_{max}$ in terms of the assumed values of C_{D_0} and K that we can use.***

Source: Fig. 3.22, Ref. PS 2 (Oates)

Best Speed, V, Estimation for Cruise and Loiter Mission Phases

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Jet Aircraft

Steady, level flight implies $L = W,\ T = D$

- **Best Loiter Time (Endurance): Minimum Drag and Minimum Thrust**

$$(L/D)_{\max} = 1/\sqrt{4C_{D0}K} \qquad C_{L\text{opt}} = \sqrt{\frac{C_{D0}}{K}} \qquad C_{D0} = KC_L^2 \qquad V_{(L/D)_{\max}} = \sqrt{\frac{2W}{\rho C_{L\text{opt}} S}} = \sqrt{\frac{2W}{\rho S}}\sqrt{\frac{K}{C_{D0}}}$$

- **Best Range Speed: Constant altitude cruise**

$$V_{\text{best range}} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{3K}{C_{D_0}}}}$$

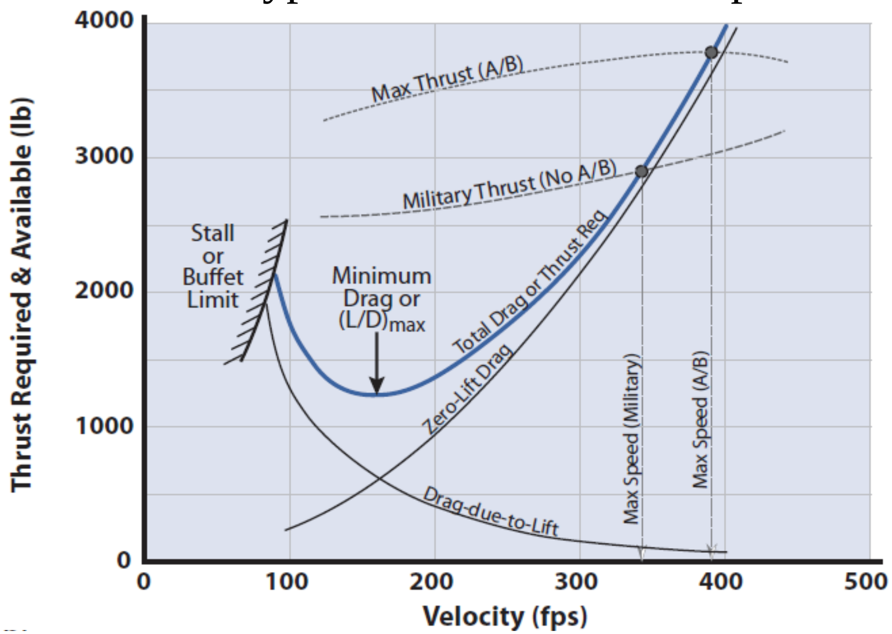
$$= 1.32\ V_{(L/D)\max}$$

- **Best Range Speed: Cruise climb**

$$V_{\text{best range}} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{2K}{C_{D_0}}}}$$

$$= 1.19\ V_{(L/D)\max}$$

Typical Jet Aircraft Example



For "Optimum" C_L, Best Speed is a Function of W/S and Altitude!

Source: Ref. AVD 1 (Nicolai and Carichner); AVD 2 (Raymer)

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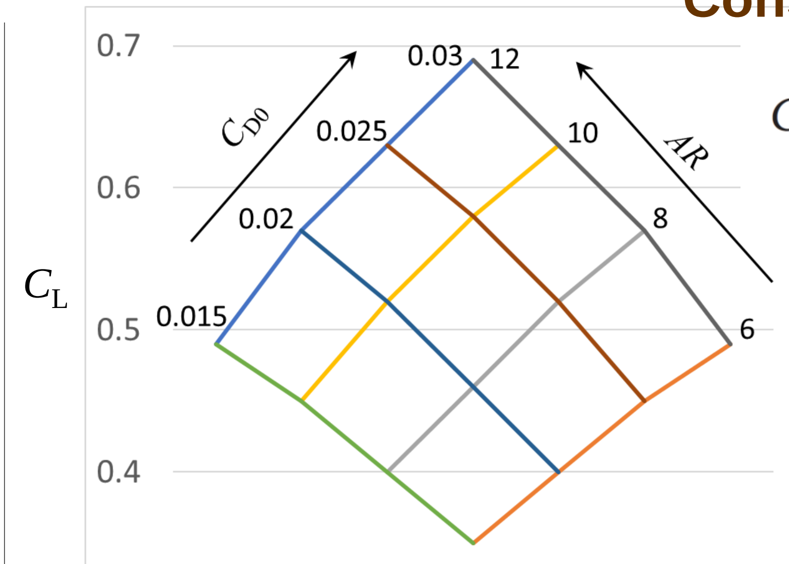
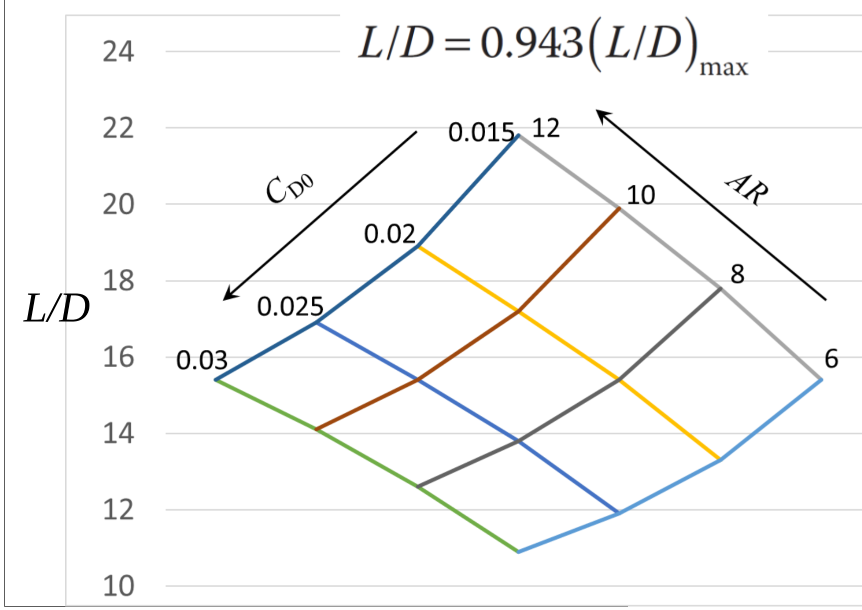
Best Range Speed: *Jet Aircraft*
Constant Throttle Cruise Climb



$$C_L = \sqrt{\frac{C_{D0}}{2K}} \qquad K = 1/(\pi A R e)$$

$$(L/D)_{max} = 1/\sqrt{4 C_{D0} K}$$

$$L/D = 0.943 (L/D)_{max}$$



Assumptions:

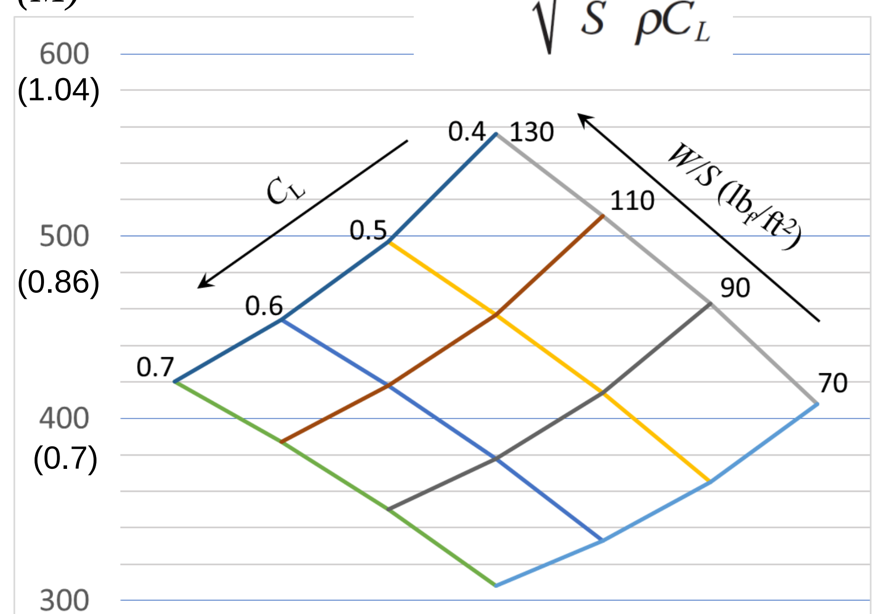
$e = 0.85$

Altitude = 35,000 ft.

Density, $\rho = 0.000773$ slugs/ft^3

Speed of sound = 579 kt

$$V = \sqrt{\frac{W}{S}\frac{2}{\rho C_L}}$$



Best Speed, V, Estimation for Cruise and Loiter Mission Phases

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Steady, level flight implies $L = W, D = T$

Propeller Aircraft

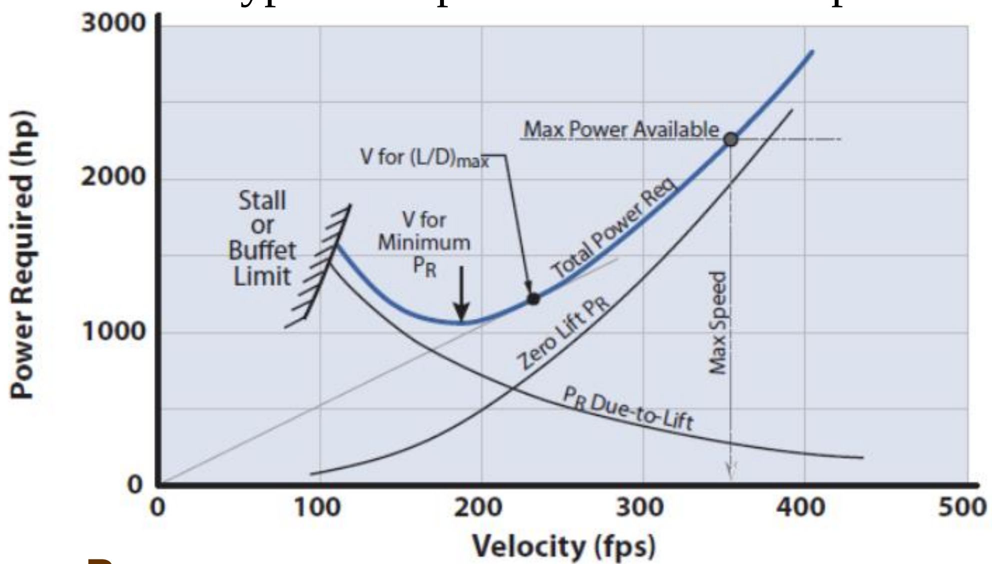
- **Best Range Speed:** **Constant altitude**

$$C_{L_{opt}} = \sqrt{\frac{C_{D_0}}{K}}$$

$$C_{D_0} = K C_L^2$$

$$V_{(L/D)_{max}} = \sqrt{\frac{2W}{\rho C_{L_{opt}} S}} = \sqrt{\frac{2W}{\rho S}} \sqrt{\frac{K}{C_{D_0}}}$$

Typical Propeller Aircraft Example



- **Best Loiter (Endurance):** **Minimum Power**

Power Required = Drag x Speed (or Velocity)

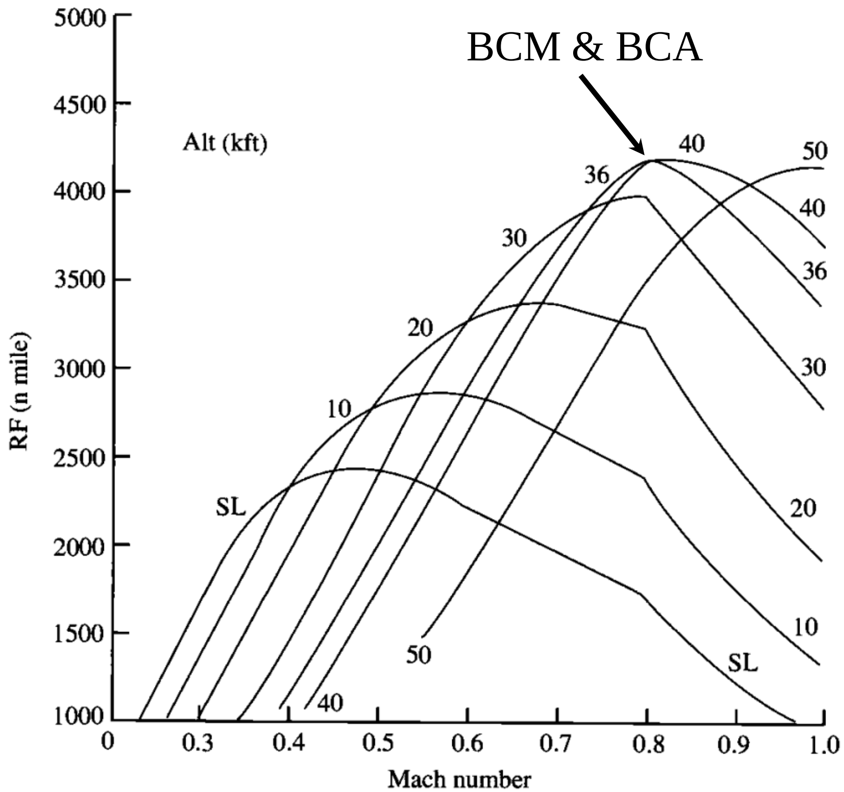
$$V_{min\,P_R} = \sqrt{\frac{2W}{\rho S}} \sqrt{\frac{K}{3 C_{D_0}}} = 0.76\, V_{(L/D)_{max}}$$

For a Prescribed C_L, Speed is a Function of W/S and Altitude!

Source: Ref. AVD 1 (Nicolai and Carichner)

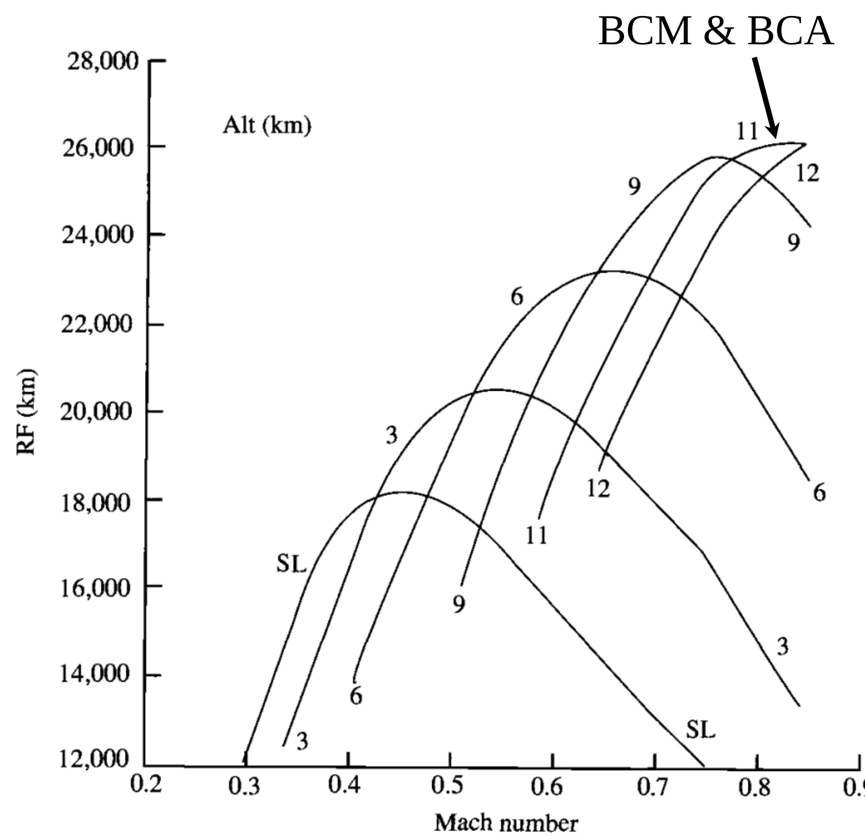
Typical Examples of Range Factor, *RF*, Variation with Mach Number, *M*, and Altitude

Notional Fighter Aircraft



low-bypass-ratio, mixed-flow turbofan with afterburner

Notional Transport Aircraft



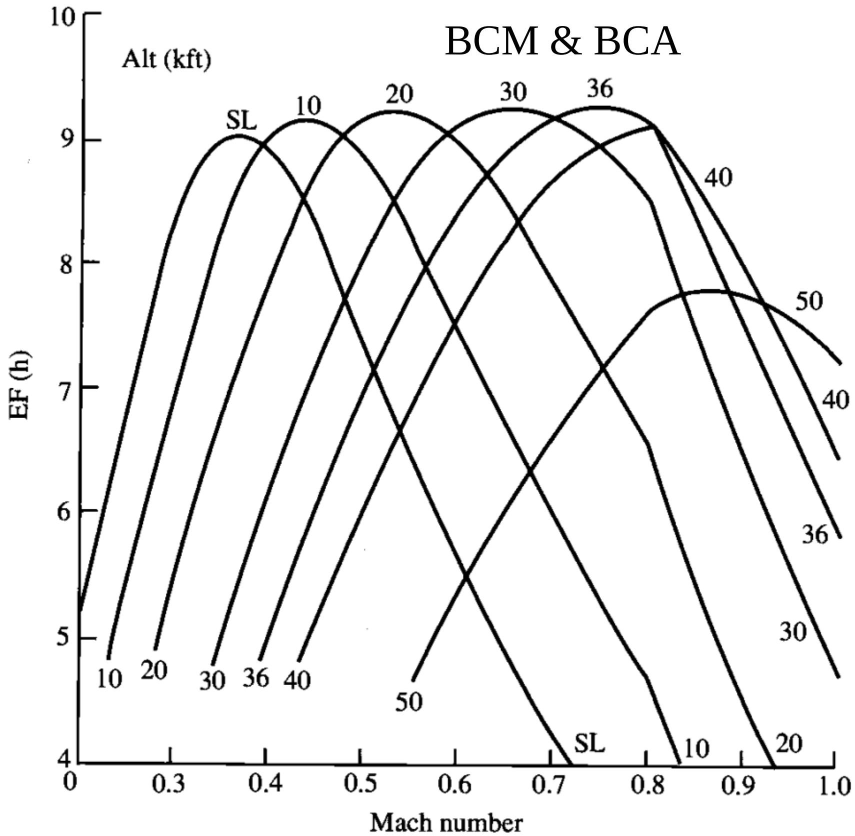
high-bypass-ratio turbofan

See Chapter 1, PS 1 (Mattingly) for details

Source: Figs. 1.31 and 1.33, Ref. PS 1 (Mattingly)

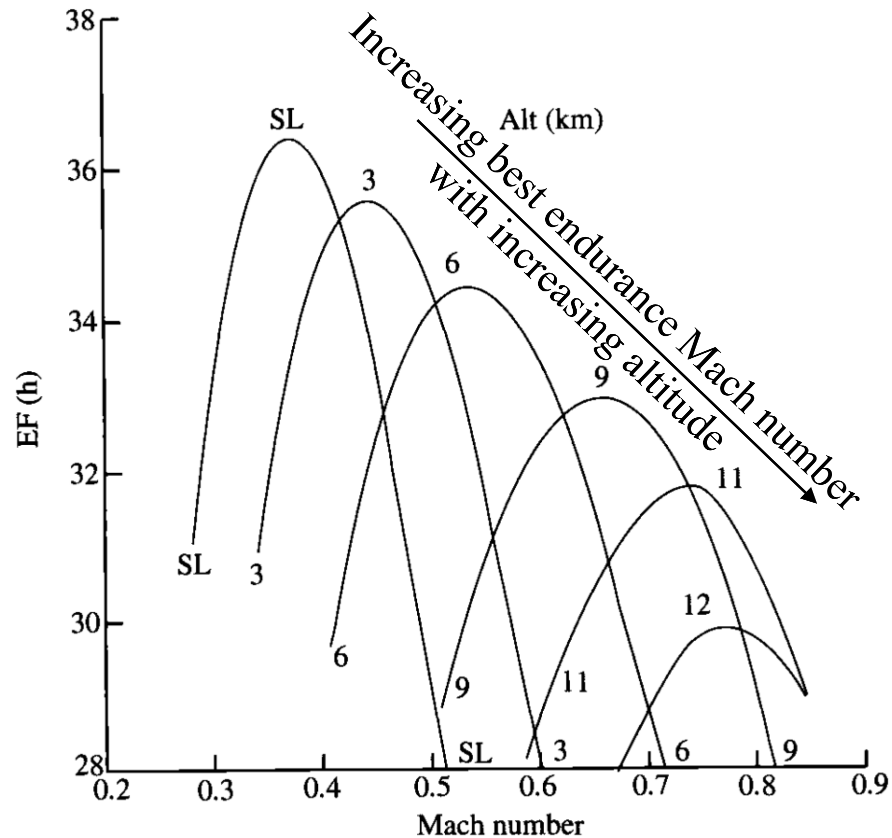
Typical Examples of Endurance Factor, EF, Variation with Mach Number, M , and Altitude

Notional Fighter Aircraft



low-bypass-ratio, mixed-flow turbofan
with afterburner

Notional Transport Aircraft



high-bypass-ratio turbofan

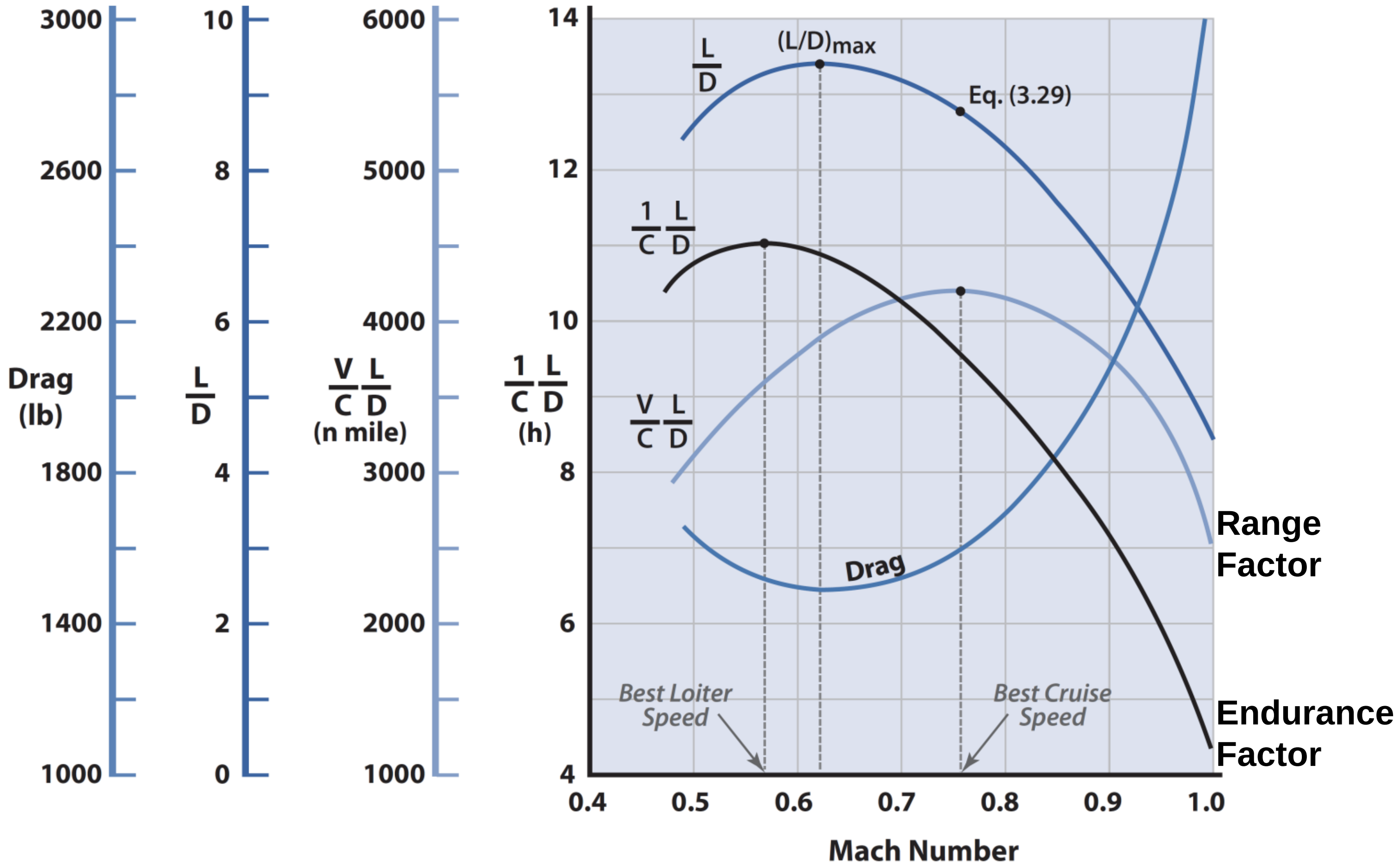
See Chapter 1, PS 1 (Mattingly) for details

Source: Figs. 1.30 and 1.33, Ref. PS 1 (Mattingly)

Best Cruise and Best Loiter Speed Example: Lightweight Fighter

Altitude = 36,000 ft.; W/S = 43 lbf/ft^2

C (= *sfc*) is a minimum at this altitude, but varies with Mach number

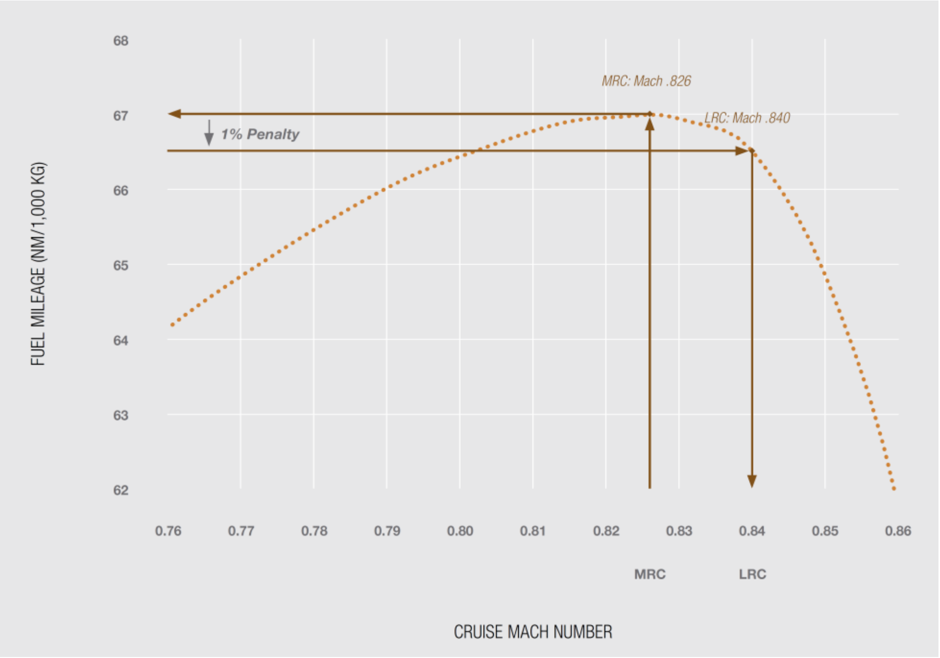


Of course, making such detailed charts requires more information (e.g., W/S) than you have at this stage of design. But you can make reasonable assumptions!

Source: Fig. 3.7 and Table 3.1, Ref. AVD 1 (Nicolai and Carichner)

Considerations Affecting Cruise Flight

- Cruising at the Maximum-Range Cruise (MRC) speed and associated altitude (the same as BCM/BCA) for minimum fuel consumption is clearly the most desirable strategy
- As shown in the figure, most long-range airliners fly at Long-Range Cruise (LRC) speed which has 1% higher fuel-consumption penalty
- The 1% loss of range is traded for 3% to 5% gain in cruise velocity!
- Today's airplanes can select an ECON speed based on a Cost Index (CI) which is the ratio of time cost ($/hr) to fuel cost (cents/lb)
- However, pilots on long-range flights often face constraints that force them to deviate from any of the initially selected cruise strategies whether MRC or ECON speed or some other
  - Speed compatible with other traffic on a specified route segment
  - Speed to achieve a required time of arrival at a point in the flight
  - Maximum endurance speed while holding
  - Directed by ATC to maintain specific speed



Source: Aero Qtly, QTR_04/07, www.boeing.com/commercial/aeromagazine

Mission Fuel Fraction Estimation (Recap)

- Use given or estimated V, and estimated L/D and sfc values, to compute W_{i+1}/W_i for Cruise and/or Loiter phases of the mission
  - **Cruise**

$$\frac{W_{i+1}}{W_i} = e^{-\frac{R \cdot sfc}{V(L/D)}}$$

  - **Endurance (Loiter)**

$$\frac{W_{i+1}}{W_i} = e^{-\frac{E \cdot sfc}{(L/D)}}$$

- For **other mission phases**, use following estimates of W_{i+1}/W_i

Warmup and takeoff	0.97
Climb & Accelerate*	0.985
Combat	*See Ref. AVD 1, pp. 130*
Descent	Part of Cruise
Landing	0.995

- Plug the estimated values into the Fuel Fraction formula

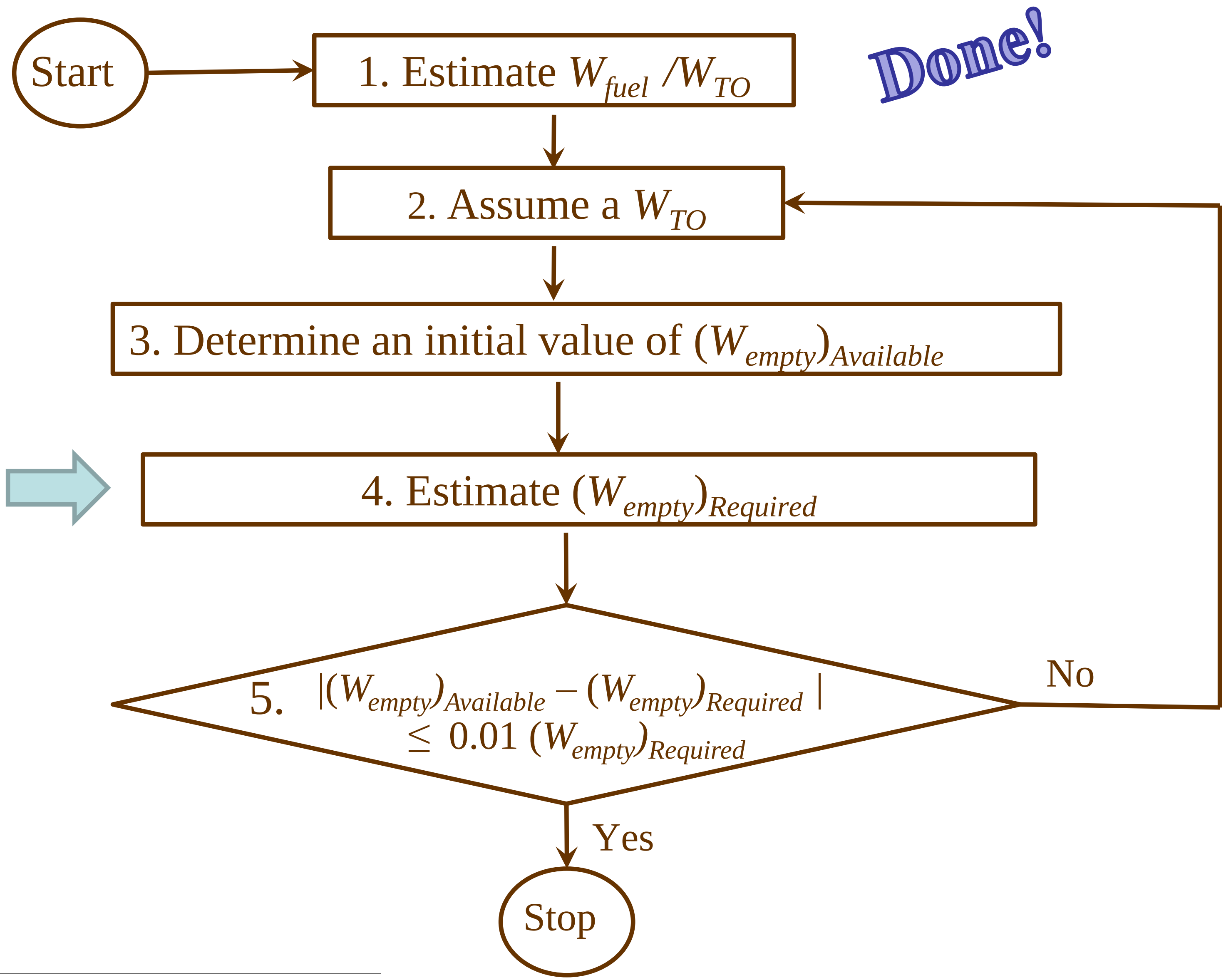
**Supersonic aircraft (Concorde, SR-71) can burn much more fuel in climb. See Fig. 5.2, Ref. AVD 1*

So we have now learned a technical approach for estimating Fuel Fraction, W_{fuel}/W_{TO}, to perform initial weight sizing!

Outline

A4. Initial Sizing: *Take-off Weight Estimation*

Flowchart of Iterative Procedure for Estimating W_{TO}



Five-step Iterative Procedure for Estimating W_{TO}

1. Estimate W_{fuel} / W_{TO} by piecemeal application of Breguet Range equation to all sections of the prescribed mission
2. Assume a value of W_{TO} (use historical data *for the class of vehicles to which your concept belongs*)
3. Determine an initial value of $(W_{empty})_{Available}$

$$(W_{empty})_{Available} = W_{TO} - W_{fuel} - W_{fixed}$$

4. Estimate $(W_{empty})_{Required}$ (use historical data *for the class of vehicles to which your concept belongs*)
5. Iterate by varying the value of W_{TO} until the following convergence criterion is met

$$\left| (W_{empty})_{Available} - (W_{empty})_{Required} \right| \leq 0.01 \, (W_{empty})_{Required}$$

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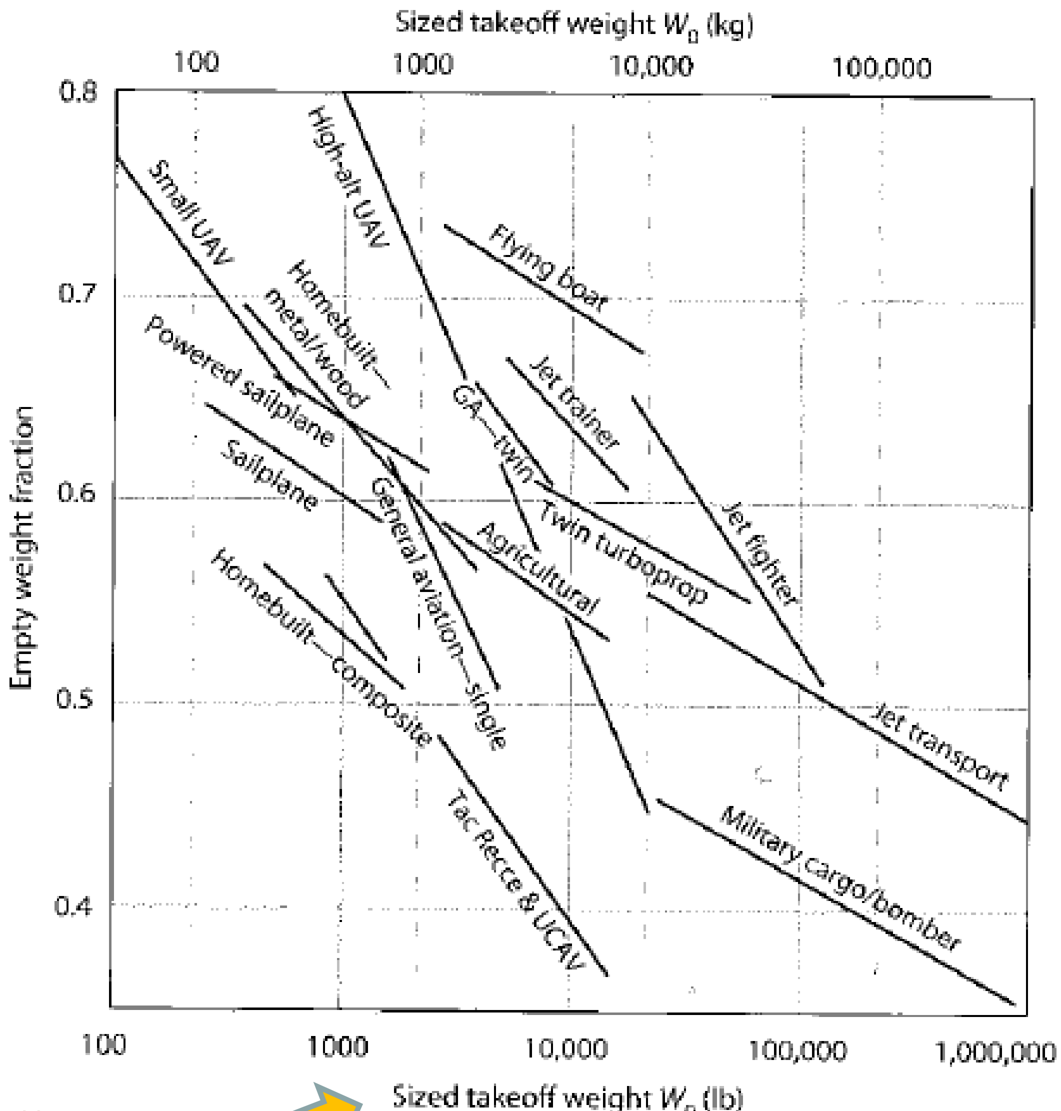
Empty Weight Fraction Estimation

Approach 1: *Use Historical Data for an assumed W_{TO}*

$(W_{empty})_{Required}$



For an assumed Takeoff Weight W_{TO}

Source: Fig. 3.1, Ref. AVD 2 (Raymer)

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Empty Weight Fraction, W_{empty}/W_{TO}, Estimation

Approach 2(a): *Use Trend Line Equations for an assumed* W_{TO}

W_{empty}/W_{TO} historical Trend Line equation $W_{empty}/W_{TO} = A(W_{TO})^C$

Aircraft Type	A	C
Sailplane—unpowered	0.86	-0.05
Sailplane—powered	0.91	-0.05
Homebuilt—metal/wood	1.19	-0.09
Homebuilt—composite	1.15	-0.09
General Aviation—single engine	2.36	-0.18
General aviation—twin engine	1.51	-0.10
Agricultural aircraft	0.74	-0.03
Twin turboprop	0.96	-0.05
Flying boat	1.09	-0.05
Jet trainer	1.59	-0.10
Jet fighter	2.34	-0.13
Military cargo/bomber	0.93	-0.07
Jet transport	1.02	-0.06
UAV—Tac Recce & UCAV	1.67	-0.16
UAV—high altitude	2.75	-0.18
UAV—small	0.97	-0.06

Source: Table 3.1, Ref. AVD 2 (Raymer)

Empty Weight, W_{empty}, Estimation

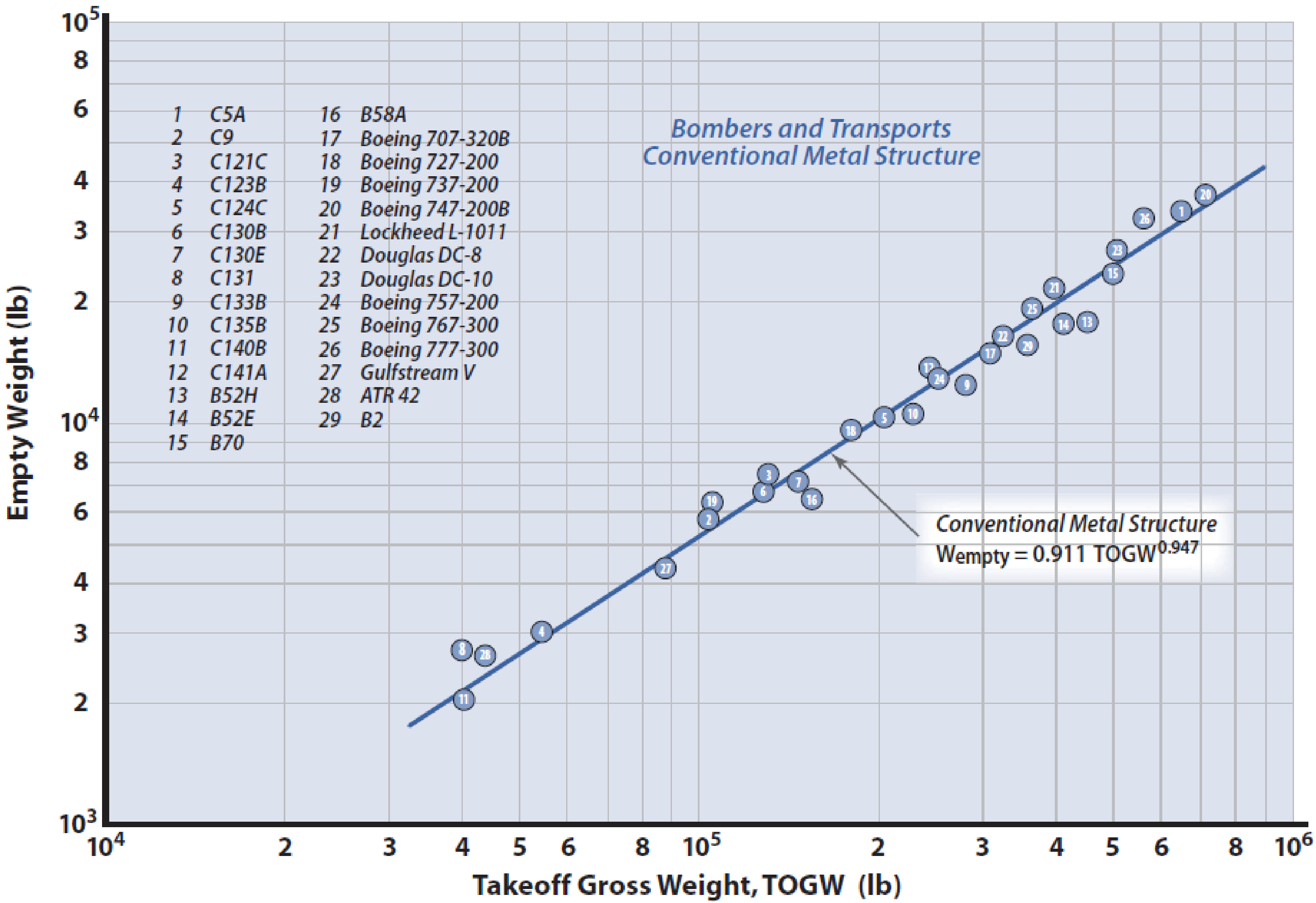
Approach 2(b): *Use Trend Line Equations for an assumed* W_{TO}

W_{empty} historical Trend Line equation $W_{empty} = (Constant)\,(W_{TO})^{XX}$

Aircraft Type	Constant	XX
Fighter		
Air-to-air or developmental	1.2	0.947
Multipurpose	0.911	0.947
Air-to-ground	0.774	0.947
Bomber and transport	0.911	0.947
Light general aviation	0.911	0.947
Composite sailplane	0.911	0.947
Military jet trainer	0.747	0.993
High altitude ISR	0.75	0.947
Unmanned air vehicles		
Propeller, endurance > 12 h	1.66	0.815
Propeller, endurance < 12 h	2.18	0.815
Turbine, ISR	2.78	0.815
Turbine maneuver UCAV	3.53	0.815
Air-launch cruise missiles and targets	1.78	0.815

Source: Table 5.1, Ref. AVD 1 (Nicolai and Carichner)

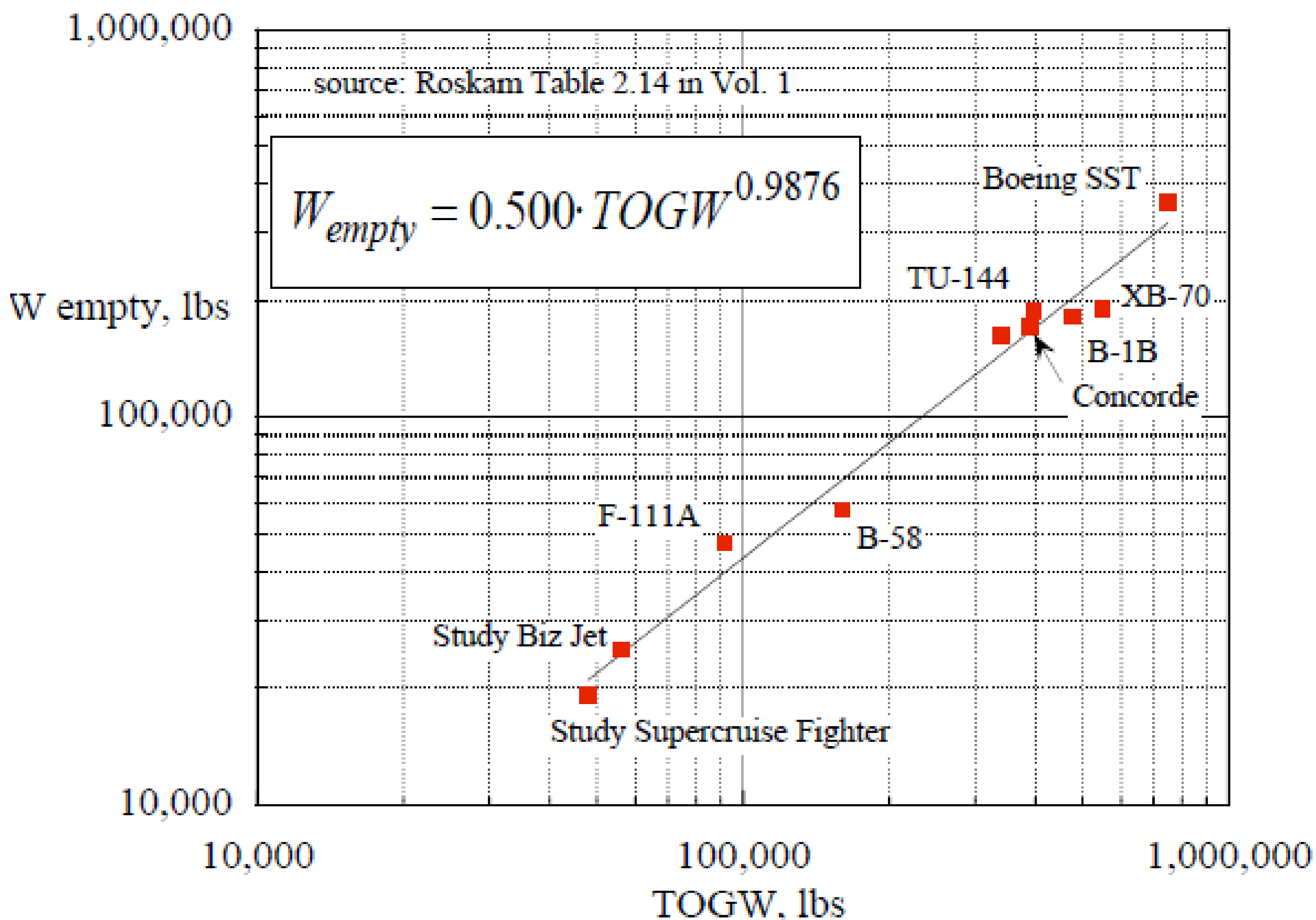
Historical Empty Weight Trend Line: ***Bombers & Transports***



- **See Appendix I (Ref. AVD 1) for other classes of aircraft**

Source: Fig. I.2, Ref. AVD 1 (Nicolai and Carichner)

Historical Empty Weight Trend Line: *Supersonic Transport*

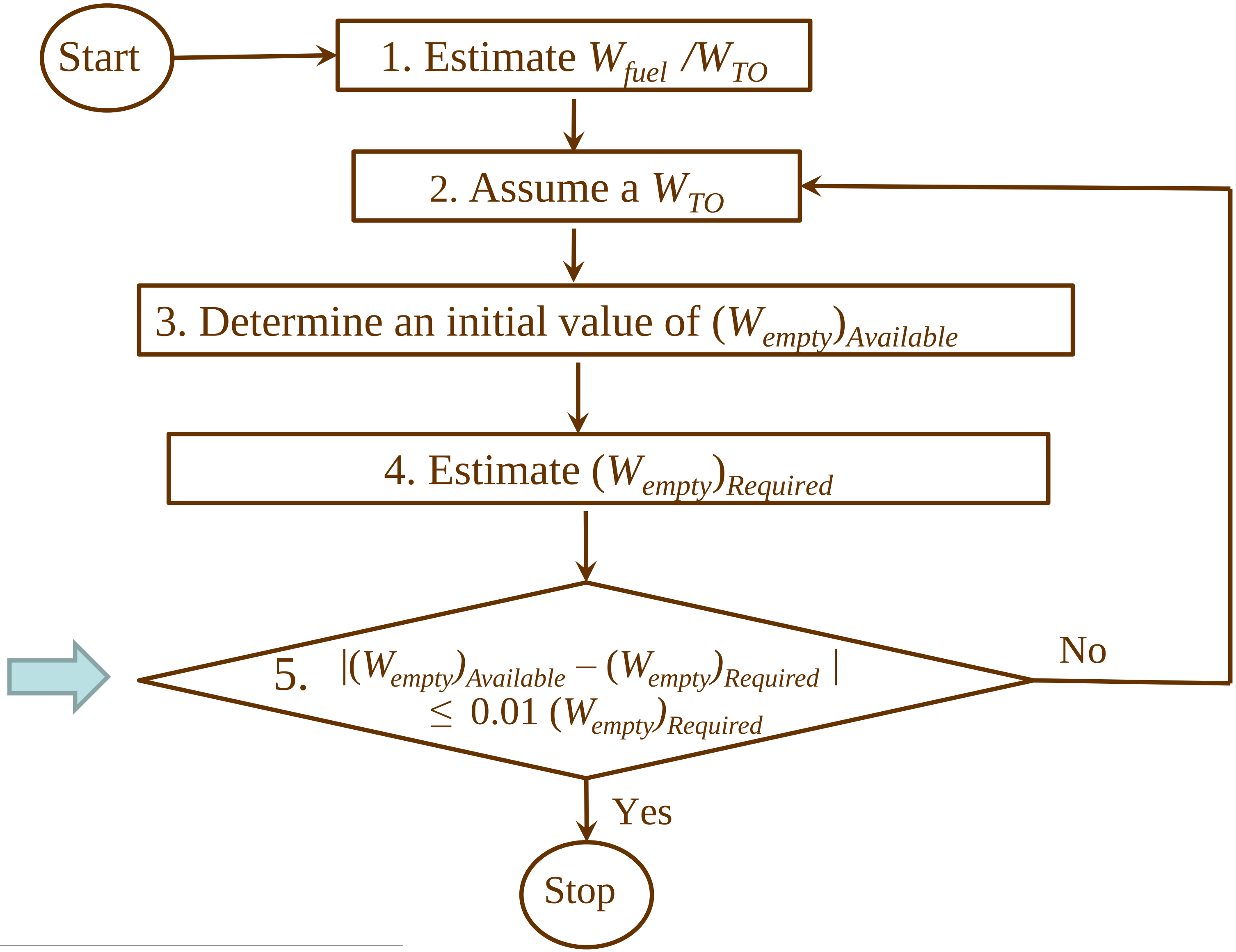


Now we know some approaches of estimating Empty Weight Fraction (Required) for Initial Weight Sizing!

Outline

A4. Initial Sizing: *Take-off Weight Estimation*

Flowchart of Iterative Procedure for Estimating W_{TO}



Five-step Iterative Procedure for Estimating W_{TO}

1. Estimate W_{fuel} / W_{TO} by piecemeal application of Breguet Range equation to all sections of the prescribed mission
2. Assume a value of W_{TO} (use historical data *for the class of vehicles to which your concept belongs*)
3. Determine an initial value of $(W_{empty})_{Available}$

$$(W_{empty})_{Available} = W_{TO} - W_{fuel} - W_{fixed}$$

4. Estimate $(W_{empty})_{Required}$ (use historical data *for the class of vehicles to which your concept belongs*)
5. Iterate by varying the value of W_{TO} until the following convergence criterion is met

$$|(W_{empty})_{Available} - (W_{empty})_{Required}| \leq 0.01\,(W_{empty})_{Required}$$

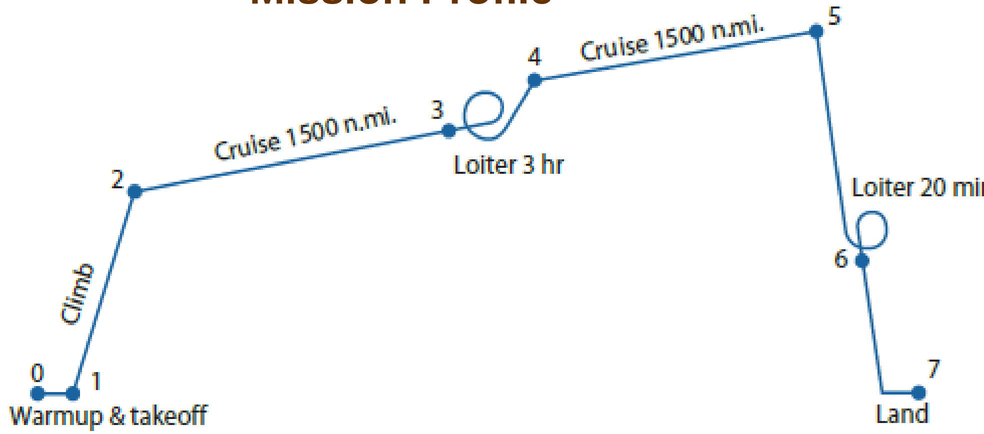
Step 5 is best illustrated using an example.

Example of TOGW Estimation:
Hypothetical ASW Aircraft

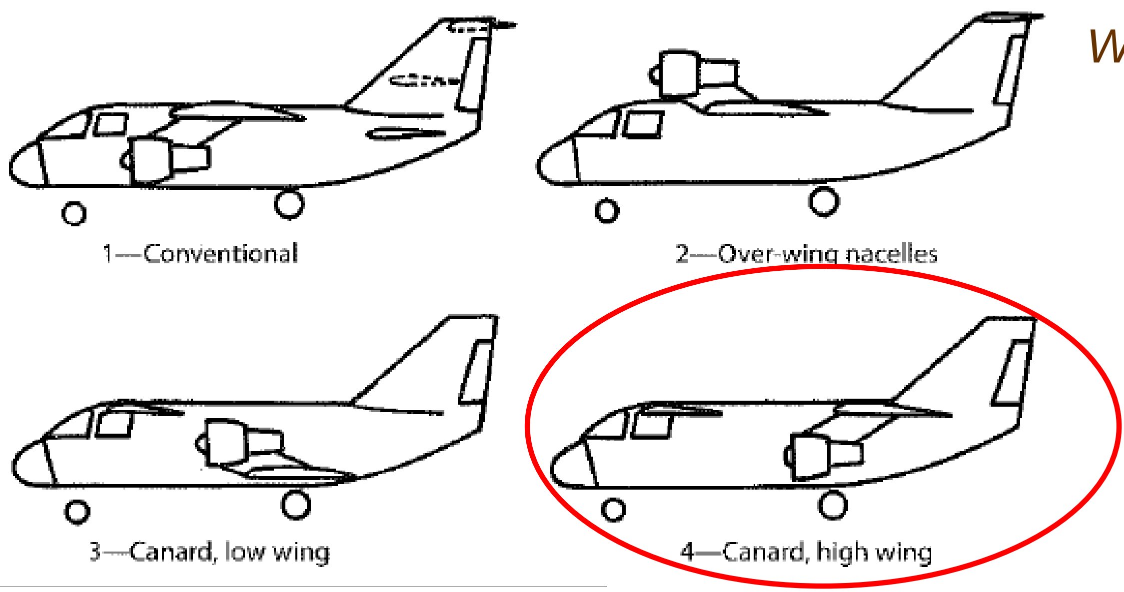
Requirements

- Loiter for three hours at 1500 nm from takeoff point, then return to base
- Cruise Mach number: 0.6
- Equipment weight: 10,000 lbs.
- Four-man crew totaling 800 lbs.

Mission Profile



Initial Configuration Sketches



How heavy is Concept 4?

What is its Takeoff Gross Weight?

$$W_{TO} = ?$$

$$W_{TO} = \frac{W_{fixed}}{1 - \frac{W_{empty}}{W_{TO}} - \frac{W_{fuel}}{W_{TO}}}$$

Given: $W_{fixed} = 10{,}800$ *lbs.*

Source: Ch. 3.6.4 , Ref. AVD 2 (Raymer)

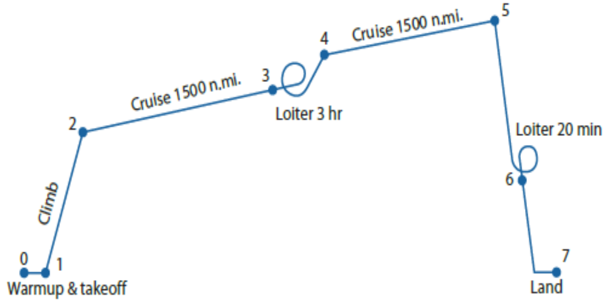
Fuel Fraction, W_{fuel} / W_{TO}, Estimation

Fuel Fraction by mission phases

Warmup and takeoff: $W_1 / W_0 = 0.97$

Climb: $W_2 / W_1 = 0.985$

Cruise (1st leg): $W_3 / W_2 = e^{\{-R*C/V(L/D)\}}$



Given: $\underline{R = 1500\ nm = 9{,}114{,}000\ ft.}$, $M = 0.6$

Assume: Typical cruise altitude = 30,000 ft.; $L/D = 0.866\ (L/D)_{max}$
High bypass ratio turbofans for efficiency
AR = 7 (Combined Wing and Canard)
$S_{wet}/S_{ref} = 5.5$

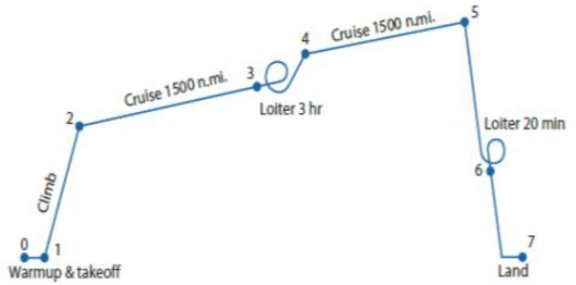
Then: *Speed of sound = 994.8 ft/s (Appendix B, Ref. 2)*
$\underline{V = 596.9\ ft/s}$
$\underline{sfc = 0.5\ lb/hr/lb = 0.000139\ lb/s/lb}$
Wetted Aspect Ratio = 7/ (5.5) = 1.27
$L/D_{max} = 16$; $\underline{L/D = 13.9}$

$$W_3 / W_2 = 0.858$$

Source: Ch. 3.6.4 , Ref. AVD 2 (Raymer)

Fuel Fraction, W_{fuel} / W_{TO}, Estimation

Fuel Fraction by mission phases (contd.)



Loiter: $W_4 / W_3 = e^{\{-E*C/(L/D)\}} = 0.9277$

Given: *E = 3 hrs = 10,800 sec*

Assume: *SFC = 0.4 lb/hr/lb = 0.000111 lb/s/lb*

$L/D = L/D_{max} = 16$

Cruise (2nd leg): $W_5 / W_4 = e^{\{-R*C/V(L/D)\}} = 0.858$

Loiter: $W_6 / W_5 = e^{\{-E*C/(L/D)\}} = 0.9917$

Given: *E = 20 min = 1,200 sec*

Assume: *SFC = 0.4 lb/hr/lb = 0.000111 lb/s/lb*

$L/D = L/D_{max} = 16$

Landing: $W_7 / W_6 = 0.995$

Mission Fuel Fraction, $W_7 / W_0 = 0.6441$

Fuel Fraction: $W_{fuel} / W_{TO} = (1 + 0.05 + 0.01)(1 - 0.6441) = 0.3773$

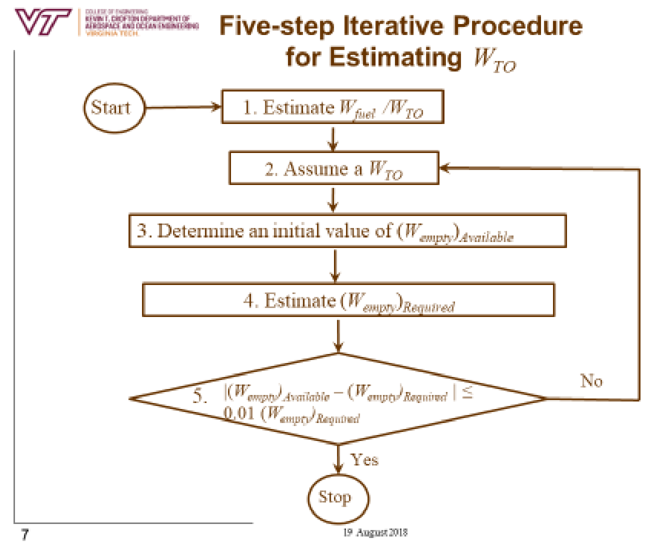
Source: Ch. 3.6.4 , Ref. AVD 2 (Raymer)

W_{TO} Estimation

$$(W_{empty})_{Available} = W_{TO} - W_{fuel} - W_{fixed}$$

$(W_{empty})_{Required}$: Using Trend Line Equation for Military Cargo/Bomber class of vehicles, we have

$$(W_{empty})_{Required} / W_{TO} = 0.93\,(W_{TO})^{-0.07}$$



Assumed W_{TO}	$(W_{empty})_{Required}$	$(W_{empty})_{Available}$	Difference
50,000	21,803	20,335	1468
60,000	25,832	27,351	1519
56,000	24,227	24,071	156
56,500	24,428	24383	45
56,700	24,508	24507	1

Is 56,000 lb_f a reasonable estimate for W_{TO}?

Source: Ch. 3.6.4 , Ref. AVD 2 (Raymer)

Sanity Check

S-3A Viking ASW Aircraft

$$W_{TO} = 52{,}539 \text{ lbs.}$$

We are in the "right ballpark"

Additional Sanity Checks

- **Check C_{D_0}**

$$(L/D)_{max} = 1/\sqrt{4C_{D_0}K}$$

  - Estimated L/D_{max} is 16
  - Assumed AR is 7; then estimated e is 0.8 and estimated $K = 1/(\pi ARe) = 0.054$
  - Estimated C_{D_0} is 0.0181 which falls within the range of historical values for high subsonic jet transport

- **Check cruise wing loading W/S**

$$V_{\text{best range}} = \sqrt{\frac{2W}{\rho S}\sqrt{\frac{3K}{C_{D_0}}}}$$

  - For the assumed cruise altitude of 30,000 ft., ρ is 0.000891 slugs/ft^3
  - Given $M_{cruise} = 0.6$, estimated W/S is 53 which falls within the range of historical values of 40 to 90 for this class of aircraft (high subsonic jet aircraft)

Remember: Iterative Procedure Gives An Answer for W_{TO} But…

- **…one pass through it does not give a Complete Answer!**
- Starting with the premise that we have documented all assumptions, the next step is to examine sensitivity of TOGW to variations in key parameters—this is called Trade Study.
- Trade Studies help answer the *"What if…"* design questions.
  - *What if avionics or fuel or cargo weight were increased by x%, how would it affect the TOGW?*
  - *What if sfc or L/D or empty weight fraction estimates were off by ±5%, ±10% or ±15%, how would it affect the TOGW?*
- Further Discussion in the Course Module on *Trade Studies*

Source: Eric Schrock (Skunk Works) Personal Communication

Outline

A4. Initial Sizing: *Take-off Weight Estimation*

A4.1 Introduction

A4.2 Aircraft with ICE Propulsion

A4.2.1 Overall Technical Approach

A4.2.2 Fuel Fraction Estimation

A4.2.3 Empty Weight Fraction Estimation

A4.2.4 Take-off Gross Weight Estimation

A4.3 Aircraft with All Electric Propulsion

A4.4 Aircraft with Hybrid Propulsion

All-Electric Propulsion Aircraft (AEA)

- **Battery-powered AEA**:
  - Goal is to assess feasibility for our AEA concept by demonstrating that it can *successfully fly to and from the destination with desired payload*.
  - Since it's reasonable to assume that AEA weight does *not* change throughout the mission, we *cannot* use the Breguet Range equation, can we?

$$R = \frac{V}{C}\frac{L}{D} ln \frac{W_{i-1}}{W_i}$$

R = Range — C = Specific Fuel Consumption (sfc)
V = Speed — L/D = Lift-to-Drag Ratio
W_i = Ending Weight — W_{i-1} = Beginning Weight

- **Cruise Range, R, of a battery-powered AEA can be estimated as**

$$R = \varepsilon_b \, (W_b/W) \, (L/D) \, \eta$$

- ✓ Cruise flight — $L = W$
- ✓ Cruise speed = Power / Drag — $V = P/D$
- ✓ Power = Energy / Time — $P = E_n/t$
- ✓ Range = Speed * Time — $R = V^*t = E_n/D = (E_n/W)\,(L/D)$
- ✓ Energy — $E_n = \varepsilon_b * W_b$
- ✓ Battery to motor shaft efficiency — η
- ✓ *Note: ε_b is battery specific energy (energy per unit mass); W_b is battery mass; W is airplane mass; and W_b/W* is Battery Mass Fraction

AEA Sizing

$$W_{TO} = \frac{W_{fixed}}{1 - \frac{W_{empty}}{W_{TO}} - \frac{W_b}{W_{TO}}}$$

- **Battery Mass Fraction, W_b/W, is *the sum* of the estimated BMFs for each of the mission segments that can be estimated using airplane performance equations much like the Range equation example on the *previous slide***
- **The iterative procedure for ICE propulsion aircraft (described in section A4.2) can be readily adapted to estimate initial AEA take-off weight**
- **Recommended sources of additional information:**
  - **Chapter 20, Ref. AVD 2 (Raymer)**
  - **Chapter 3, Ref. AVD 3 (Gundlach)**

Outline

A4. Initial Sizing: *Take-off Weight Estimation*

Hybrid-Electric Propulsion (HEP) Aircraft

- **Active area of research**
  - HEP offers many more options for design and operations to consider than either ICE or AE aircraft.
  - Goal is to assess feasibility of our concept by demonstrating that the design can *successfully fly to and from the destination with desired payload.*
  - One approach for Initial Weight estimation is presented here. It's adapted from: Cetracchio, F. et al, "Approach to the Weight Estimation in the Conceptual Design of Hybrid-Electric-Powered Unconventional Regional Aircraft," *Journal of Advanced Transportation*, 2018 (https://doi.org/10.1155/2018/6320197)

- **For a typical hybrid-electric propulsion system with two sources of energy, namely, fuel and battery, we can express takeoff weight as**

$$W_{TO} = W_{fixed} + W_{empty} + W_{fuel}^{H} + W_{battery}^{H}$$

$$W_{TO} = \frac{W_{fixed}}{1 - \frac{W_e}{W_{TO}} - \frac{W_f^H}{W_{TO}} - \frac{W_b^H}{W_{TO}}}$$

- **If we could estimate fuel weight fraction and battery weight fraction, we can use the five-step iterative procedure to estimate** W_{TO}

Hybrid-Electric Propulsion Aircraft

- **Two "Degree of Hybridization" Parameters**

1. **Power:** Degree of Hybridization for power, $H_P = \frac{P_{em}}{P_{total}} = \frac{P_{em}}{P_{em} + P_{ICE}}$
   - P_{em} is maximum installed power of electric motor
   - P_{ICE} is installed power of internal combustion engine

2. **Energy:** Degree of Hybridization for energy, $H_E = \frac{E_{elec}}{E_{total}} = \frac{E_{elec}}{E_{elec} + E_{fuel}}$
   - E_{elec} is electric energy
   - E_{fuel} is fuel energy
   - H_E is needed because H_P does not account for energy storage

- **Fuel Weight Fraction**
  - Can be estimated much like for ICE (see Section A4.2.2)
  - Assuming 5% reserve and 1% trapped fuel, fuel fraction for a N segment mission, can be expressed as

$$\frac{W_f^H}{W_{TO}} = 1.06\left[1 - \prod_{i=1}^{N}\left\{\frac{W_{i+1}}{W_i} + \left(1 - \frac{W_{i+1}}{W_i}\right)(H_P)_i\right\}\right]$$

  - $(H_P)_i$ is the degree of hybridization for the *i-th* mission segment

Hybrid-Electric Propulsion Aircraft

- **Battery Weight Fraction**
  - Assuming the hybridization factor for power, H_P, to be a constant for all mission segments, we can estimate the battery weight fraction using energy balance considerations as:

$$\frac{W_b^H}{W_{TO}} = \left(\frac{\varepsilon_f}{\varepsilon_b}\right)\left(\frac{\eta_{conv}^{th}}{\eta_{conv}^{elec}}\right)\left(\frac{H_P}{1 - H_P}\right)\left(\frac{W_f^H}{W_{TO}}\right)$$

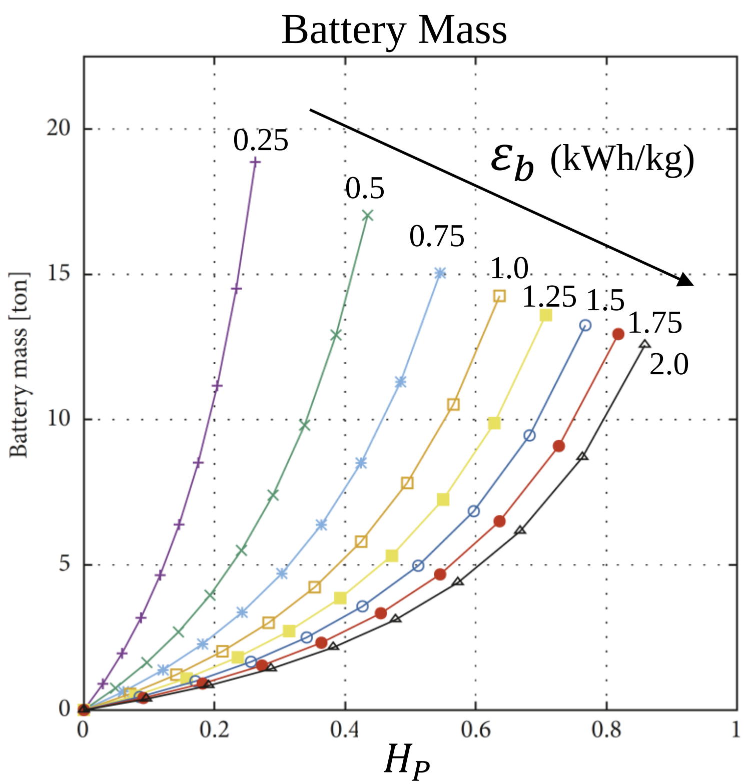
    - H_P is the degree of hybridization for all mission segments
    - ε_f is the fuel specific energy (energy per unit mass)
    - ε_b is the battery specific energy
    - η_{conv}^{th} is the efficiency of converting fuel energy into power

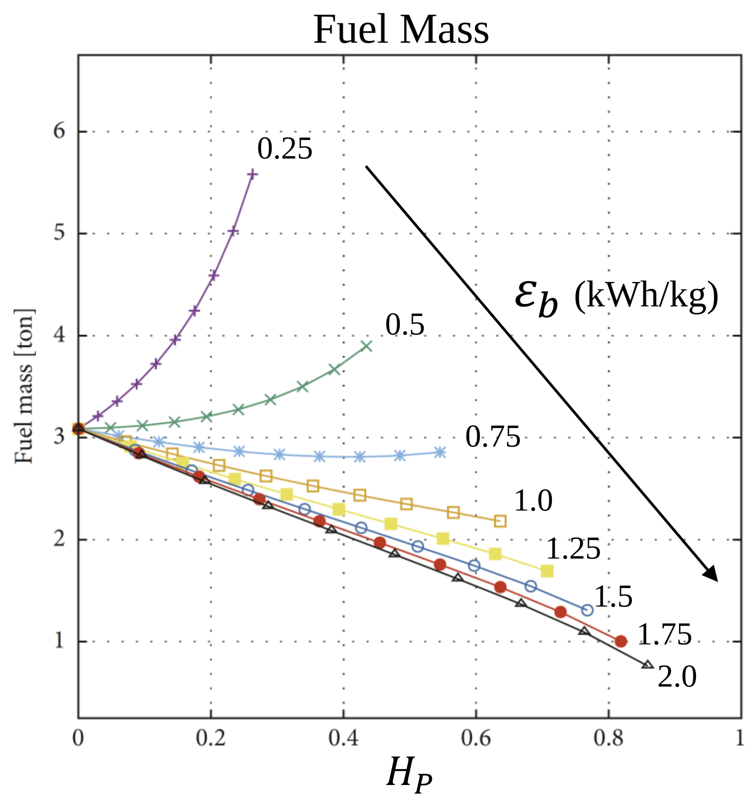
      For ICE engines, typically value is 0.3 to 0.35
    - η_{conv}^{elec} is the efficiency of converting battery energy into power

      For electric motors, typical value is 0.85 to 0.9
  - Notice that higher battery specific energy is desirable for low battery weight!
    - Today's batteries have ε_b = 250 Wh/kg

Battery and Fuel Mass Sensitivity to Hybridization Parameter, H_P

- For a regional aircraft with Range = 900 nm; Payload = 100 pax + 5 crew; Cruise Mach number = 0.5; Cruise altitude = 25,000 ft; Cruise L/D = 19; and Cruise BSFC = 211 g/kWh; the following charts illustrate sensitivity of battery and fuel mass to H_P





Source: Cetracchio, F. et al, "Approach to the Weight Estimation in the Conceptual Design of Hybrid-Electric-Powered Unconventional Regional Aircraft," *Journal of Advanced Transportation*, 2018 (https://doi.org/10.1155/2018/6320197)

Epilogue

Now that we know how heavy the airplane is, next we need an initial estimate of

(i) wing area required to produce the needed lift, and

(ii) total thrust required to propel the aircraft

so that the aircraft can successfully perform the mission while meeting all requirements and regulations.

Recommended Readings

Ref. No.	Chapter	Author(s)	Title
AVD 1	**Chapter 5**	Nicolai, L.M. and Carichner, G.E.	***Fundamentals of Aircraft and Airship Design , Volume I—Aircraft Design ,*** AIAA Education Series, AIAA, Reston, VA, 2010.
AVD 2	**Chapters 3, 6, 20**	Raymer, D.P.	***Aircraft Design : A Conceptual Approach ,*** AIAA Education Series, AIAA, Reston, VA, 2012.
AVD 5	**Chapter 4**	Sadrey, M.H.	***Aircraft Design: A Systems Engineering Approach ,*** John Wiley & Sons, Inc., 2013.

NOTE: See Appendix in the Overview CM